

Competitive Dynamics in Organizations

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Abstract

Political economy has studied how institutions divide an already-produced surplus, but less how that surplus is politically produced. We develop a political theory of production in which organizational performance depends on how superiors secure political control within hierarchies. Strong legal enforcement reduces reliance on personal loyalty and sustains performance-based competition. Under weak enforcement, however, authority capital, the superior's loyal following, can substitute for legal institutions by allowing political control and intense competition to coexist. Productivity therefore declines only when both legal enforcement and authority capital are weak, helping explain performance variation within and across regime types. The same mechanism generates political inequality through loyalty-based differences in effort. Finally, we show that greater authority capital lowers the institutional tipping point at which weaker legal enforcement becomes privately attractive, revealing how organizational political control can erode incentives to preserve formal institutions.

Keywords: authority capital; legal enforcement; political control; performance competition; organizational productivity; institutional stability

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Introduction

Political economy offers many theories explaining why economic performance varies within and across regime types. For instance, work on electoral competition and constitutional design explains variation in the composition of public spending and the extent of rent extraction (Persson et al., 2000); work on clientelism explains why patronage crowds out efficient public provision even under robust electoral competition (Robinson and Verdier, 2013); and selectorate theory links the public–private mix of goods provision to the size of a leader’s winning coalition (Bueno de Mesquita et al., 2003).

These theories have provided valuable insights into how political institutions shape economic outcomes. Yet they largely approach economic outcomes through the politics of redistribution: how an already-produced surplus is divided among public goods, patronage, targeted transfers, or rents. They tell us much about how political institutions “allocate the cake”, but little about how “the cake is politically produced” before the redistribution can be realized. To understand how political mechanisms shape productivity, we need not only a politics of distribution, but also a political theory of production: one that treats economic performance as an *ex ante* incentive problem rooted in the political conditions of production inside organizations. How, then, do political and institutional conditions shape the incentives through which production is organized?

Production requires not only economic incentives but also political control capable of sustaining compliance within the organizations where production takes place (Coase, 1937; Simon, 1951; Miller, 1992; Bowles and Gintis, 1993; Burawoy, 1985). Yet political control does not sustain itself. Across organizations, from states and bureaucracies to firms, a common way for superiors to secure it is to place loyal subordinates in key positions. In a hierarchical structure where subordinates far outnumber key positions, promotion typically creates internal competition based on relative performance. Favoritism toward loyalists, however, may condition this competition, alter subordinates’ effort, and ultimately shape organizational output. This paper formalizes this tension in a model of competitive dynamics across institutional contexts.

The model is set in a principal-agent framework that extends Berry’s (1993) contest framework to incorporate political control. A superior allocates a valuable position among subordinates who are equally capable but differ in loyalty, defined as their propensity to comply with the superior’s directives once promoted. Subordinates exert costly effort to compete for the position, while the superior determines the intensity of performance-based competition.

The extent to which organizational control depends on loyalty varies with the institutional environment. Where the rule of law is weak, formal institutions provide limited protection against opportunistic behavior and cannot reliably enforce coordina-

tion within organizations. A superior who delegates control over organizational assets or authority to a subordinate therefore faces greater risks that the subordinate may appropriate those assets, disregard her directives, or otherwise act against her interests. Personal loyalty becomes an important substitute for formal enforcement. The weaker the rule of law, the more the superior must rely on loyal subordinates to maintain control and coordinate the organization. Where the rule of law is stronger, property rights, enforceable contracts, and legal institutions perform more of these functions, reducing the superior's dependence on loyalty.

A separate consideration is how easily the superior can secure such loyalty-based control. This depends on her authority capital, defined as the number of subordinates already loyal to her. Greater authority capital increases the likelihood that a loyal subordinate ultimately obtains the position, making it easier for the superior to secure political control through loyalty. Together, the superior's dependence on loyalty and the ease with which loyalty-based control can be secured determine how intensely she runs the competition for the position and, consequently, how much effort the competition elicits from subordinates.

Organizational productivity remains high through two distinct mechanisms, when either legal enforcement or the superior's authority capital is strong. When legal enforcement is strong, the superior relies less on personal loyalty to secure compliance and can therefore sustain promotion based primarily on relative performance. More meritocratic competition strengthens subordinates' incentives to exert productive effort, generating higher aggregate output. When legal enforcement is weak but authority capital is high, intense competition can be sustained for a different reason. Because loyalists constitute a large share of the subordinate pool, a meritocratic winner is already likely to be loyal. The superior can therefore maintain intense performance-based competition without substantially jeopardizing political control. High authority capital thus allows productive competition and political control to coexist even where formal institutions are weak.

Organizational productivity declines only when legal enforcement and the superior's authority capital are both weak. Under these conditions, the superior cannot rely on either formal institutions or an established network of loyal subordinates to secure compliance. She therefore relaxes performance-based competition and shifts promotion away from merit and toward loyalty. This shift weakens subordinates' incentives to exert productive effort, ultimately reducing aggregate organizational output.

The model therefore highlights a source of organizational instability that formal institutions alone cannot capture: leadership may change while the rules remain the same, yet performance can shift because the incoming leader commands a different stock of authority capital.

Political control also generates inequality in productive effort. When favoritism

enters promotion, loyal subordinates exert more effort than disloyal subordinates because they face better prospects of obtaining valuable positions or resources. These politically generated effort gaps need not move systematically with aggregate productivity: similar levels of output can coexist with very different levels of effort inequality.

In the model extension, we examine how authority capital shapes the superior's incentives toward legal institutions. We identify an institutional tipping point beyond which weaker legal enforcement becomes privately preferable to the strong-law benchmark. As authority capital increases, this tipping point falls, expanding the range of legal environments in which institutional weakening becomes privately attractive. The result identifies an organizational mechanism through which the political incentives to preserve strong formal institutions can erode even before those institutions themselves change.

This paper contributes to three fields in political economy. First, our theory offers a new perspective on productivity differences within and across political regimes. Rather than treating regime type as the operative source of variation, we disaggregate the political foundations of productivity into two more proximate conditions: the strength of legal enforcement and the superior's authority capital. Because these conditions need not map one-to-one onto broad regime categories, organizations operating under the same regime type may sustain different levels of performance competition, while organizations across different regime types may generate similar productive incentives. Indeed, an organization in a weaker legal environment can sustain higher output than one in a stronger legal environment when its superior commands more authority capital (Proposition 4). The model may therefore help account for mixed empirical findings on the relationship between regime type and economic performance by identifying institutional and organizational variation that broad regime categories leave unobserved.

Second, this paper offers a different perspective on the relationship between economic performance and inequality. The classic Kuznets hypothesis links development systematically to income inequality, with inequality first rising and then falling as development proceeds (Kuznets, 1955, 1966). Acemoglu and Robinson (2002) politicize this relationship by showing how different political trajectories can generate different development–inequality paths. Our model shifts attention to a more fundamental source of inequality: politically generated differences in productive effort. Favoritism creates systematic effort gaps between loyal and disloyal subordinates, yet these gaps bear no stable relationship to aggregate productivity. Both high and low productivity can coexist with either high or low effort inequality.

Third, our theory identifies an organizational source of institutional instability. Existing work documents the gradual weakening of institutional constraints (Bermeo, 2016) and develops theories of when incumbents engage in democratic backsliding (Grillo et al., 2024). We shift attention to a prior question: what determines the in-

cumbent's private valuation of strong institutions in the first place? Our answer lies in the organizational foundations of political control. As authority capital increases, reliance on loyalty becomes politically less risky, reducing the superior's private attachment to strong legal enforcement and lowering the institutional tipping point. Formal institutions can therefore become vulnerable even without an initial change in those institutions, because the political resources that sustain actors' incentives to preserve them have changed.

The paper proceeds as follows. Section discusses related literature. Section presents the baseline model. Section examines how authority capital shapes the superior's preference over legal strength. Section discusses implications for productivity, inequality, and institutional stability. Section concludes.

Related Literature

Our paper builds on the organizational-economics literature on contests, tournaments, and promotion incentives. Classic tournament theory studies how relative performance evaluation and prize structures shape agents' incentives to exert effort (Tullock, 1980; Lazear and Rosen, 1981; Rosen, 1986; Skaperdas, 1996), while subsequent work develops contest success functions for environments with multiple prizes (Nalebuff and Stiglitz, 1983; Berry, 1993). Related work in personnel economics emphasizes promotion and career concerns as mechanisms for both eliciting effort and selecting high-performing employees within organizations (Gibbons and Murphy, 1992; Prendergast, 1999; Baker et al., 1994). This literature treats the competitive structure itself as fixed by productive considerations, leaving its political determinants outside the analysis. We instead endogenize the competitive structure as an organizational choice constrained by the superior's need to maintain political control, and examine how the resulting competition shapes productive effort and organizational output.

A closely related literature studies the trade-off between loyalty and competence in political appointments. Egorov and Sonin (2011) show that rulers may sacrifice competence for loyalty because more competent advisers are better able to identify profitable opportunities for defection. Zakharov (2016) derives a similar trade-off from subordinates' outside options: more competent officials have better prospects under alternative rulers and therefore weaker incentives to invest in the incumbent's survival. More recently, Dessein and Garicano (2025) embed the loyalty–competence trade-off in political tournaments, showing how competition can increase leaders' demand for loyal team members who are less likely to exit when political prospects deteriorate. Despite their different mechanisms, these accounts share a common focus: loyalty affects organizational performance primarily through the selection and retention of personnel

with different levels of competence or talent.

Our framework shifts attention from the selection effect of loyalty to its incentive effect. We abstract from the loyalty–competence trade-off by allowing subordinates to be equally capable and instead ask how political considerations shape the competition through which they are selected. In particular, competition is not simply an exogenous environment in which leaders choose between loyal and talented agents; its intensity is itself an endogenous organizational choice shaped by the superior’s need for political control. Although promotion determines who ultimately occupies the position, our central mechanism operates before selection occurs: favoritism toward loyalists changes the expected return to productive effort, thereby altering how intensely all subordinates compete and, ultimately, how much the organization produces. Loyalty can therefore impact organizational productivity even when it entails no sacrifice in the competence of the person selected.

Finally, our paper speaks to theories of institutional change. A large literature explains institutional transformation through shifts in the distribution of political power (Acemoglu et al., 2005) and through critical junctures that unsettle existing institutional arrangements (Collier and Collier, 1991; Capoccia and Kelemen, 2007). Other work emphasizes that institutional change can arise endogenously, without any external disruption. Greif and Laitin (2004), for example, show how institutions can generate processes that gradually alter the parameters sustaining their own equilibrium, while theories of gradual institutional change emphasize how ambiguities in institutional rules and their implementation create opportunities for internally generated transformation (Mahoney and Thelen, 2010).

Our framework identifies a distinct source of endogenous incentives for institutional change: the organizational foundations of political control. Whereas Helmke and Levitsky (2004) focus on how formal and informal institutions interact, we show how variation in informal political control can reshape actors’ incentives toward formal institutions themselves. Authority capital can therefore affect not only how political control operates under a given institutional environment, but also the superior’s incentive to preserve that environment.

The Model

Superiors who preside over hierarchies rarely command compliance by fiat. A party leader cannot force local cadres to implement policy faithfully; a bureau chief cannot compel careerist subordinates to execute directives against their own interest; a firm’s CEO cannot simply order loyalty from division heads who control day-to-day operations. Compliance has to be secured, and it can be secured in essentially two ways.

Where courts, audits, and enforceable employment contracts are credible, a superior can rely on formal rules: promotion tracks performance, and legal or bureaucratic enforcement backstops the arrangement even against subordinates who would rather shirk or defect once in office. Where such enforcement is weak, the superior has no choice but to substitute an informal guarantee for a formal one: she places people she already trusts, because of kinship, faction, or personal loyalty, into the positions from which compliance is ultimately extracted.

The difficulty is that loyalty and performance-based competition pull against one another. Favoring loyalists secures the superior's political control, but it blunts the very competition that elicits effort from everyone else. This section builds a model to trace exactly how a superior resolves that tension, and to show that her resolution depends on two political conditions: the strength of the legal environment surrounding the organization, and the size of the loyal following she has already built. Three ingredients do the work. First, subordinates differ only in loyalty, not in ability, and the superior can act on loyalty only within whichever pool of top performers she has already committed to considering, so favoritism operates *through*, not instead of, competition. Second, the weight the superior places on loyalty is disciplined by the surrounding rule of law. Third, that weight interacts with how large a loyal following, or *authority capital*, the superior already commands. We formalize each in turn, characterize the resulting equilibrium, and then ask how the superior optimally designs the competition itself.

Set Up

Organizational Structure and Agent Types. Consider a hierarchical organization consisting of a superior, S , and $n \geq 2$ subordinates. The superior might be a political leader allocating provincial appointments, a bureaucratic chief controlling promotion within an agency, or a firm's CEO choosing who reports directly to her; in each case she presides over more capable people than she has positions to reward. The subordinates compete for a single indivisible prize, $V > 0$, such as a promotion or a delegation of authority. The superior's central design choice is the rule that governs who is eligible to receive it.

Subordinates are identical in productive ability and in the cost of exerting effort, but they differ along a dimension that matters for political control rather than for output: loyalty, defined as a subordinate's personal willingness to comply with the superior's directives once given delegated authority. We refer to subordinates of the two resulting types as loyal (l) and disloyal (d). Of the n subordinates, $m \in \{0, 1, \dots, n\}$ are loyal and the remaining $n - m$ are disloyal; we call m the superior's *authority capital*, since it indexes the loyal following she can already draw on and, other things equal, a larger

m leaves her political position more secure. Both individual types and m are common knowledge; m is treated as exogenous throughout.

Legal strength and the value of loyalty. Let $\alpha \geq 0$ measure how much the superior's political control depends on whom she promotes, which is decreasing in the strength of the rule of law. Law is uniform and impersonal: it binds every office-holder by the same rules, irrespective of personal allegiance. Where legal enforcement is strong ($\alpha = 0$), the law itself secures compliance, so the identity of the promoted subordinate is politically inconsequential and both types yield the superior the same impersonal benchmark, which we set to zero. As legal enforcement weakens, the law can no longer secure compliance on its own, and whom the superior promotes becomes increasingly consequential for her control. The weakening of law is itself impersonal. It withdraws the same guarantee from every office-holder rather than from any particular type, and therefore raises the importance of the promotion decision by a single common magnitude, α . Relative to the impersonal benchmark, promoting a loyal subordinate secures voluntary compliance and yields $+\alpha$, whereas promoting a disloyal subordinate yields disobedience and $-\alpha$. The symmetry of these payoffs thus follows from the impersonality of law: α measures how much it matters who is promoted, and that stake is the same whichever way the promotion goes. The authority term $\alpha(P_L - P_D)$ is accordingly the superior's expected political return from the promotion relative to the strong-law benchmark. Like m , α is treated as exogenously given in the baseline model.

The Eligibility Rule. Before subordinates act, the superior sets the rule governing who can receive the prize: an eligibility threshold

$$k \in \mathcal{K} \equiv \{1, \dots, n\}.$$

After effort is exerted, only the k highest-ranked subordinates are eligible to receive the prize. We refer to the inverse of k as *competition intensity*, since performance-based competition is more intense the smaller k is.

At one extreme, $k = 1$ is a strict tournament: only the highest-ranked subordinate is eligible, leaving the superior no discretion over who receives the prize. At the other extreme, $k = n$ makes every subordinate eligible regardless of performance, severing promotion from performance entirely. Intermediate values $1 < k < n$ preserve a performance requirement while still leaving the superior some latitude over who, among the qualified, actually receives the prize.

This threshold is the model's central political-design variable. A narrow eligible pool disciplines subordinates through pure competition; a broad one leaves the su-

perior room to reward loyalty instead. The trade-off between the effort a competition elicits and the political control it affords is the mechanism the remainder of this section formalizes.

Stochastic Favoritism. Favoritism in this model cannot simply override the competition: a superior who wants to reward loyalty can act on it only within whichever pool of top performers she has already committed to considering, or her professed reliance on performance becomes transparently hollow. Concretely, the final recipient is selected from the top- k eligible set as follows. If that set contains at least one loyal subordinate, the superior selects uniformly among the loyal subordinates in it: if exactly $q \in \{1, \dots, \min(m, k)\}$ loyal subordinates rank among the top k , each receives the prize with conditional probability $1/q$, and every eligible disloyal subordinate receives it with probability zero. If the top- k set contains no loyal subordinate, the superior selects uniformly among the k eligible disloyal subordinates, so each receives the prize with conditional probability $1/k$.

Favoritism is therefore conditional on performance rather than independent of it. A loyal subordinate cannot receive the prize without first entering the top- k set, and a disloyal subordinate is not mechanically excluded from promotion: a disloyal recipient prevails whenever every eligible subordinate happens to be disloyal. Because the composition of the top- k set depends probabilistically on subordinates' effort choices, the advantage loyalty confers is itself stochastic. Formally, let $P_{li}(x_{li}, \mathbf{x}_{-i}; k)$ and $P_{dj}(x_{dj}, \mathbf{x}_{-j}; k)$ denote the probability that loyal subordinate i and disloyal subordinate j , respectively, receive the prize, and let $P_L(\mathbf{x}; k) \equiv \sum_{i=1}^m P_{li}(\mathbf{x}; k)$ and $P_D(\mathbf{x}; k) \equiv \sum_{j=1}^{n-m} P_{dj}(\mathbf{x}; k)$ denote the resulting probabilities that the prize goes to a loyal or a disloyal subordinate; since exactly one prize is awarded, $P_L(\mathbf{x}; k) + P_D(\mathbf{x}; k) = 1$.

Timing

The sequence of the game is as follows. First, the organizational and institutional parameters (n, m, V, α) are realized and commonly observed. Second, the superior chooses the eligibility threshold k . Third, subordinates simultaneously choose effort, which determines the top- k eligible set through relative performance. Finally, the superior awards the prize according to the stochastic-favoritism rule above.

We solve the game by backward induction: for each k , we first characterize a type-symmetric Nash equilibrium of the subordinate effort subgame, and then determine the superior's optimal eligibility rule given these equilibrium responses.

The Superior

The superior's problem is to set k so as to balance two things she cares about: how much the organization produces, and whether power ultimately rests with someone she trusts. For any threshold k , let $x_l^*(k; m)$ and $x_d^*(k; m)$ denote the resulting type-symmetric effort levels, $E^*(k; m) \equiv mx_l^*(k; m) + (n - m)x_d^*(k; m)$ aggregate output, and $P_L^*(k; m)$ and $P_D^*(k; m)$ the resulting equilibrium probabilities that the prize goes to a loyal or disloyal subordinate. Anticipating these responses, the superior's payoff from choosing k is

$$U_S(k; \alpha, m) = E^*(k; m) + \alpha [P_L^*(k; m) - P_D^*(k; m)] = E^*(k; m) + \alpha [1 - 2P_D^*(k; m)], \quad (1)$$

where the second equality uses $P_L^*(k; m) + P_D^*(k; m) = 1$. The superior then chooses an eligibility rule

$$k^*(\alpha, m) \in \arg \max_{k \in \mathcal{K}} U_S(k; \alpha, m).$$

The first term in (1) is the superior's productive stake in the outcome; the second is its expected authority value relative to the strong-law benchmark, capturing how much the eventual promotion reinforces or erodes her political control.

Subordinates

Subordinates, for their part, face an ordinary career-competition problem. After observing k , each simultaneously chooses effort $x \geq 0$ to maximize the value of the prize, $V > 0$, net of a quadratic effort cost, taking the resulting promotion probability as given: loyal subordinate $i \in \{1, \dots, m\}$ solves $\max_{x_{li} \geq 0} \{P_{li}(x_{li}, \mathbf{x}_{-i}; k)V - x_{li}^2/2\}$, and disloyal subordinate $j \in \{1, \dots, n - m\}$ solves the identical problem with P_{dj} in place of P_{li} . The two types differ only in the promotion probabilities the superior's rule assigns them, not in ability, effort costs, or how much they value the prize. Any equilibrium gap between loyal and disloyal effort is therefore a product of the promotion rule itself, not of an assumed difference in talent or ambition between political insiders and outsiders.

Equilibrium Effort and Organizational Output

We now characterize the subordinate effort subgame for a fixed eligibility threshold k : first the probability that the prize is ultimately awarded to a loyal or disloyal subordinate, then equilibrium effort, and finally the resulting aggregate organizational output.

Winning Probabilities

Under stochastic favoritism, a disloyal subordinate can win only if every member of the top- k eligible set happens to be disloyal, and whether that is even possible depends on how many disloyal subordinates the organization has. If $n - m \geq k$, there are enough disloyal subordinates to fill the entire top- k set, so a disloyal subordinate wins with positive probability whenever disloyal subordinates exert positive effort. If $n - m < k$, by contrast, every feasible top- k set must contain at least one loyal subordinate, and because the superior selects a loyal subordinate whenever one is eligible, disloyal subordinates are then shut out of the prize entirely.

Writing x_l and x_d for the type-symmetric effort levels, the type-level probability that the prize is awarded to a disloyal subordinate is

$$P_D(x_l, x_d; k) = \begin{cases} \frac{(n-m) \binom{n-m-1}{k-1} x_d}{\binom{n-1}{k-1} [mx_l + (n-m)x_d]}, & n - m \geq k, \\ 0, & n - m < k, \end{cases} \quad (2)$$

with $P_L(x_l, x_d; k) = 1 - P_D(x_l, x_d; k)$. Equation (2) captures the central political asymmetry the promotion rule creates: disloyal subordinates keep a positive chance at the prize only so long as it remains feasible for them to occupy every position in the eligible set. Once k exceeds $n - m$, that possibility disappears, and the superior's preference for loyalty forecloses it entirely.

Equilibrium Effort

For each k , we focus on a type-symmetric Nash equilibrium of the subordinate effort subgame.¹ When $n - m \geq k$, both types can win, and combining their first-order conditions yields an equilibrium relationship,

$$x_l^*(k; m) = B(k, m, n) x_d^*(k; m), \quad B(k, m, n) \geq 1, \quad (3)$$

that any interior type-symmetric equilibrium must satisfy; the expression for $B(k, m, n)$ and the underlying first-order conditions are in Appendix . Without presenting the closed-form solutions, we can already characterize how favoritism orders effort across types.

Lemma 1 (Equilibrium Effort under Stochastic Favoritism). *Suppose $1 \leq m \leq n - 1$. For every $k \in \{1, \dots, n\}$, the subordinate effort subgame has a unique type-symmetric equilibrium, and in it loyal subordinates exert weakly more effort than disloyal sub-*

¹A type-symmetric equilibrium requires symmetry within, but not across, subordinate types: $x_{l_i}^*(k; m) = x_l^*(k; m)$ for every $i = 1, \dots, m$ and $x_{d_j}^*(k; m) = x_d^*(k; m)$ for every $j = 1, \dots, n - m$. Thus $x_l^*(k; m)$ need not equal $x_d^*(k; m)$.

ordinates:

$$x_l^*(k; m) \geq x_d^*(k; m).$$

More specifically:

1. If $k = 1$, then $x_l^*(1; m) = x_d^*(1; m) > 0$.
2. If $1 < k \leq n - m$, then $x_l^*(k; m) > x_d^*(k; m) > 0$.
3. If $n - m < k < n$, then $x_l^*(k; m) > 0 = x_d^*(k; m)$.
4. If $k = n$, then $x_l^*(n; m) = x_d^*(n; m) = 0$.

Individual payoffs are strictly concave in own effort, so first-order conditions characterize best responses; the proof of Lemma 1 is in Appendix . The four cases trace out exactly how much room stochastic favoritism has to operate. At $k = 1$, only the single top performer is eligible, so the superior has no discretion left to exercise once performance is realized: loyal and disloyal subordinates face identical odds and exert identical effort. Once $1 < k \leq n - m$, favoritism can operate within the eligible set: a loyal subordinate who makes the cut is prioritized in the final allocation, which raises the marginal return to loyalist effort above that of disloyal effort, though disloyal effort remains positive because an all-disloyal eligible set is still possible. Once k exceeds $n - m$, every feasible eligible set necessarily contains a loyalist, so a disloyal subordinate has no path to the prize and rationally stops competing altogether, even as loyal subordinates keep exerting effort to win priority among themselves. Finally, at $k = n$ eligibility is universal and effort no longer affects anything, so both types optimally exert zero effort.

Aggregate Organizational Output

Aggregate organizational output, $E^*(k; m) = mx_l^*(k; m) + (n - m)x_d^*(k; m)$ as defined above, is the total effort exerted by all subordinates. Although stochastic favoritism redistributes effort between loyal and disloyal subordinates, that redistribution does not by itself change the total amount of effort the competition elicits, once the eligibility rule is held fixed.

Proposition 1 (Aggregate Effort Conditional on the Eligibility Rule). *For any fixed $k \in \{1, \dots, n\}$, equilibrium aggregate effort is*

$$E^*(k; m) = \sqrt{V \frac{n - k}{k}}. \quad (4)$$

Conditional on k , aggregate effort is independent of authority capital m and of the superior's reliance on loyalty α . It strictly decreases in k and equals zero at $k = n$: a more restrictive eligibility rule induces greater aggregate effort.

Proposition 1 separates the incentive effect of the eligibility rule from the distributional effect of favoritism. A larger k broadens the eligible set and weakens the link between relative performance and promotion, so subordinates face a lower marginal return to effort and aggregate output falls; at the limit $k = n$, promotion is entirely decoupled from performance and aggregate effort vanishes. Authority capital m , by contrast, changes only how a given amount of effort is *distributed* across loyal and disloyal subordinates for a fixed k : loyal subordinates exert more, disloyal subordinates less, and the two changes exactly offset each other in the aggregate.

This conditional invariance does not make m or α irrelevant to organizational output. Both remain exogenous in the baseline model, but they shape the superior's choice of the eligibility rule $k^*(\alpha, m)$ itself, and it is through that choice, rather than through any direct effect at a fixed k , that authority capital and reliance on loyalty end up shaping realized output $\hat{E}(\alpha, m) \equiv E^*(k^*(\alpha, m))$. The remainder of this section characterizes that institutional-choice channel. The proof of Proposition 1 is in Appendix .

Figure 1 illustrates the patterns in Lemma 1 and Proposition 1: loyal effort remains positive over a wider range of eligibility rules than disloyal effort, because relative performance keeps mattering among loyalists even after disloyal subordinates have been shut out; and aggregate effort declines as k grows, but at any fixed k is invariant to authority capital m , which reshuffles the composition of effort without changing its total.

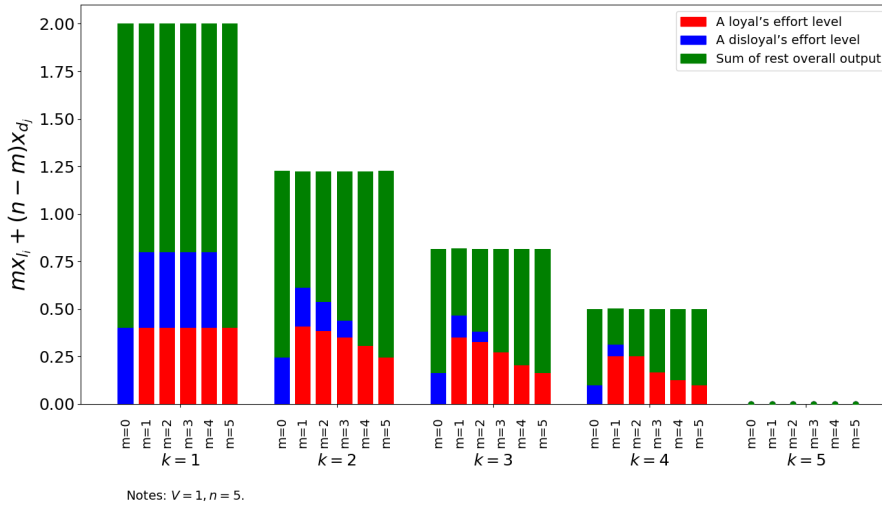


Figure 1 Equilibrium Effort and Aggregate Output Conditional on the Eligibility Rule

The Optimal Choice of Competition

Having characterized the subordinate effort subgame, we turn to the superior's choice of k itself. A smaller k buys more intense, performance-based competition; a larger k buys more room for loyalty to shape the outcome. For notational convenience, let $A(k; m) \equiv 1 - 2P_D^*(k; m)$ denote the expected authority value of the promotion out-

come under rule k , and, using (1) together with the fact that aggregate effort $E^*(k) = \sqrt{V(n-k)/k}$ does not depend on m for fixed k , write the superior's problem as choosing k to maximize $E^*(k) + \alpha A(k; m)$ over $\mathcal{K} = \{1, \dots, n\}$. The superior's institutional choice is therefore a straightforward trade-off: a smaller eligible set raises aggregate output, a larger one raises the scope for using loyalty to determine the final recipient.

Pairwise Institutional Comparisons

Because k is discrete, this trade-off resolves rule by rule. Consider two rules $k' > k$. Moving from k to k' costs the superior $E^*(k) - E^*(k') > 0$ in aggregate output, but it can gain her authority value $\alpha[A(k'; m) - A(k; m)]$ whenever the broader pool raises the odds that a loyalist ultimately wins. Whenever $A(k'; m) > A(k; m)$, there is accordingly a reliance-on-loyalty threshold

$$\hat{\alpha}_{k,k'}(m) \equiv \frac{E^*(k) - E^*(k')}{A(k'; m) - A(k; m)} \quad (5)$$

above which the superior prefers the broader rule k' to k , and below which she prefers k . Because her overall optimum is the upper envelope of these pairwise comparisons across all $k \in \mathcal{K}$, some intermediate rules may never be optimal for any α ; the optimal rule can jump directly from, say, $k = 1$ to $k = 3$ without $k = 2$ ever being chosen. Except at the finite set of α where two rules tie, the optimal rule $k^*(\alpha, m)$ is unique.

Lemma 2 (Benchmark without reliance on loyalty). *When $\alpha = 0$, the superior chooses the strict tournament, $k^*(0, m) = 1$ for every m .*

With no political stake in who wins ($\alpha = 0$), the superior maximizes aggregate output alone, and since $E^*(k)$ is strictly decreasing in k , the strict tournament is her unique optimum. This benchmark isolates the pure productivity motive before the two political forces of interest, reliance on loyalty and authority capital, are allowed to operate.

Reliance on Loyalty and the Optimal Rule

Expanding the eligible set raises the probability that loyalty can shape the final outcome, so the expected authority value to a broader rule is weakly increasing in α .

Proposition 2 (Reliance on Loyalty and Competition). *For any given authority capital m , the optimal eligibility threshold $k^*(\alpha, m)$ is weakly increasing in the superior's reliance on loyalty α : $\alpha' \geq \alpha \implies k^*(\alpha', m) \geq k^*(\alpha, m)$. Equivalently, weaker legal institutions induce weakly less intense performance-based competition.*

The logic is monotonic and intuitive: for any two rules $k' > k$, the relative payoff of the broader rule rises with α , so once the superior prefers k' at some level of reliance on

loyalty, a further increase in α can never reverse that preference. Politically, stronger legal institutions reduce the superior's need to lean on loyal subordinates to secure compliance, so she places relatively more weight on aggregate output and sustains a tighter, more performance-based rule. As legal institutions weaken, loyalty becomes correspondingly more valuable, and a broader eligibility rule becomes increasingly attractive.

Authority Capital and Institutional Choice

Authority capital's effect is more nuanced, because it operates only through how it changes the marginal authority gain from broadening the eligible set. For any $k' > k$, define this gain as

$$G_{k,k'}(m) \equiv A(k'; m) - A(k; m). \quad (6)$$

Because aggregate output at a fixed k does not depend on m , authority capital shapes the superior's institutional choice only by moving $G_{k,k'}(m)$, and depending on how it moves, three distinct institutional logics can arise.

Proposition 3 (Authority Capital and Institutional Regimes). *The effect of authority capital on the optimal eligibility rule can be characterized as follows.*

1. **Authority-capital neutrality.** *In a neighborhood of $\alpha = 0$, $k^*(\alpha, m) = 1$ for every feasible m : changes in authority capital leave the superior's optimal rule unchanged.*
2. **Discretion activation.** *An increase in authority capital can instead broaden the optimal rule. If the optimal rule is unique at both m and $m + 1$ and $k^*(\alpha, m + 1) = k' > k = k^*(\alpha, m)$, then*

$$\alpha G_{k,k'}(m) < E^*(k) - E^*(k') < \alpha G_{k,k'}(m + 1),$$

so the additional loyalist raises the marginal authority gain from broadening, $G_{k,k'}(m + 1) > G_{k,k'}(m)$. Such switches can arise only where the condition in part 3 fails.

3. **Authority–discretion substitution.** *If $G_{k,k'}(m + 1) \leq G_{k,k'}(m)$ for every pair $k < k'$ in $\mathcal{K}$, then $k^*(\alpha, m + 1) \leq k^*(\alpha, m)$ for every $\alpha \geq 0$, for both the smallest and the largest optimal rule when the optimum is not unique: greater authority capital weakly substitutes for promotion discretion and supports a more restrictive rule. The condition holds if and only if the increment $A(k; m + 1) - A(k; m)$ is weakly decreasing in k (Lemma A.6).*

Part 1 follows from Proposition A.5 and part 3 from Proposition A.4 in Appendix ; the inequalities in part 2 follow directly from the optimality of k at m and of k' at $m + 1$, and Proposition A.7 shows that such switches occur.

Figure 2 summarizes these three regions schematically; because α measures reliance on loyalty, low α corresponds to strong legal institutions and high α to weak ones.

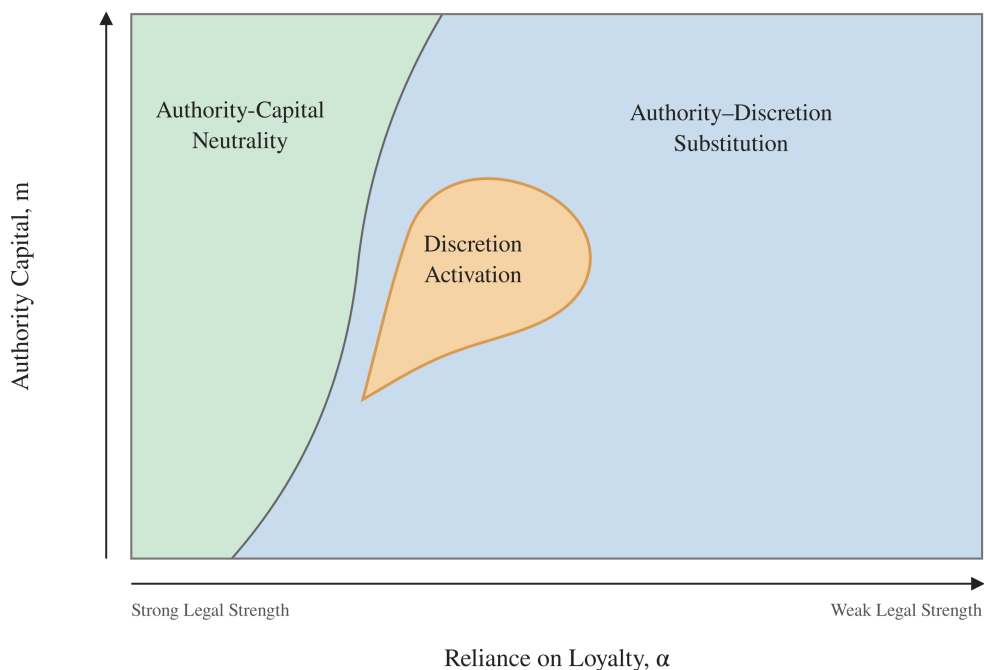


Figure 2 Authority Capital, Promotion Discretion, and Institutional Regimes

The three regions correspond to three distinct political situations. Where legal institutions are strong and reliance on loyalty is correspondingly low, the productive advantage of the strict tournament simply dominates the authority benefit of any broader rule: authority capital may still affect who ends up winning, but it does not change the superior's optimal k . Where the loyalty network starts out thin, adding a single loyalist can activate promotion discretion for the first time: if the resulting authority gain crosses the threshold needed to offset the output it costs, the superior broadens the pool. This can happen only where the increment in authority value from an additional loyalist rises with the breadth of the eligible set. And where the superior already commands a substantial loyal following, an additional loyalist instead substitutes for discretion: she can restore a tighter, more productive rule precisely because political control no longer depends on keeping the pool broad. The relationship between authority capital and competition intensity is therefore not globally monotonic. Whenever the increment in authority value from an additional loyalist is weakly decreasing in the breadth of the eligible set, however, more authority capital either leaves the optimal

rule unchanged or makes it weakly more restrictive.

Figure 3 illustrates the global solution numerically for $n = 5$ and $V = 1$; darker cells indicate a smaller optimal $k^*(\alpha, m)$ and therefore higher output.

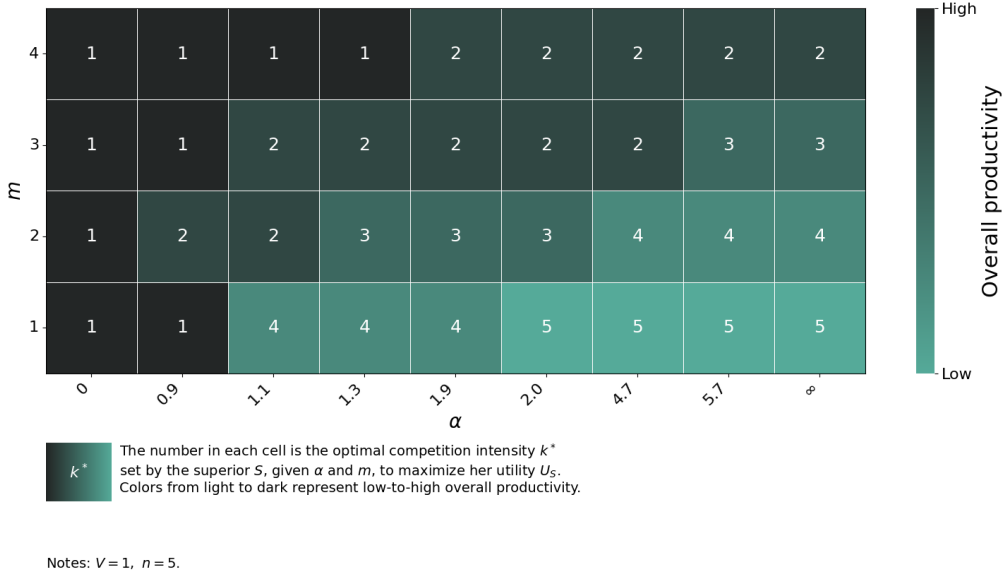


Figure 3 Optimal Eligibility Rules across Reliance on Loyalty and Authority Capital

The heat map illustrates all three regimes at once. Holding m fixed, the optimal rule is weakly increasing in α , consistent with Proposition 2. In most cells of this example, increasing authority capital leaves k^* unchanged or lowers it, the authority–discretion substitution effect. A localized pocket also exists in which increasing m instead raises k^* . This pocket is the discretion-activation region, where the extra loyalist shifts the global upper envelope of the superior’s payoff functions enough to make the broader rule newly attractive. Figure 3 reports the exact global solution for this numerical example, while Figure 2 distinguishes the underlying political logic operating in each region of the parameter space.

Corollary 1 (Induced Productivity Effects). *Because aggregate equilibrium output is strictly decreasing in k , the institutional comparative statics above imply the following productivity effects:*

1. *Aggregate output is weakly decreasing in the superior’s reliance on loyalty α .*
2. *In the authority-capital-neutrality region, changes in m leave aggregate output unchanged.*
3. *In the discretion-activation region, an increase in m may reduce aggregate output by inducing a broader eligibility rule.*
4. *In the authority–discretion-substitution region, an increase in m weakly raises aggregate output by supporting a more restrictive eligibility rule.*

The upshot is that weak legal institutions alone do not determine organizational productivity. What matters is how reliance on loyalty interacts with the superior's existing stock of authority capital: a thin loyalty network can push her to relax performance-based competition, while a sufficiently dense one lets her restore a stricter rule instead. The next result makes this precise by comparing environments that differ in both dimensions.

Proposition 4 (Productivity across Legal and Organizational Environments). *For every $n \geq 3$ and $V > 0$, there exist two environments (α_L, m_L) and (α_H, m_H) with $\alpha_H > \alpha_L$ and $m_H > m_L$ such that*

$$k^*(\alpha_H, m_H) < k^*(\alpha_L, m_L) \quad \text{and} \quad \hat{E}(\alpha_H, m_H) > \hat{E}(\alpha_L, m_L).$$

Proposition 4 shows that an organization embedded in a weaker legal environment can nonetheless sustain more intense performance competition and higher output than one embedded in a stronger legal environment, provided that its superior commands sufficiently more authority capital. The construction behind the result is transparent. With a single loyal subordinate, a strict tournament places the prize in disloyal hands with high probability, so even moderate reliance on loyalty leads the superior to broaden eligibility. With only one disloyal subordinate, a strict tournament already delivers a loyal winner with high probability, so the superior retains it even under greater reliance on loyalty. Proposition 2 therefore orders productivity across legal environments only when authority capital is held fixed. Because legal strength and authority capital need not move together across organizations or regimes, legal strength alone does not order productivity across them. The proof is in Appendix .

Political Inequality

Recall that aggregate equilibrium output is

$$E^*(k) = mx_l^*(k; m) + (n - m)x_d^*(k; m) = \sqrt{V \frac{n - k}{k}}. \quad (7)$$

For a given eligibility rule k , aggregate output is independent of authority capital m . By contrast, politically induced effort inequality is generally determined jointly by the eligibility threshold and the composition of the subordinate population:

$$I^*(k; m) = x_l^*(k; m) - x_d^*(k; m).$$

This separation implies that aggregate productivity and the distribution of effort across subordinate types are distinct organizational outcomes.

Proposition 5 (Separation of Productivity and Political Inequality). *For fixed n and V , aggregate equilibrium output is determined by the eligibility threshold k , whereas politically induced effort inequality is determined jointly by k and authority capital m . Consequently:*

1. *Political inequality can arise at every interior productivity level generated by an intermediate eligibility rule $k \in \{2, \dots, n-1\}$.*
2. *Holding k fixed, variations in authority capital can alter the equilibrium effort gap without changing aggregate output.*
3. *There is no globally monotonic relationship between aggregate productivity and politically induced effort inequality.*

The first part of the proposition follows because every intermediate eligibility rule produces both a positive effort gap and an interior level of aggregate output:

$$I^*(k; m) > 0 \quad \text{and} \quad 0 < E^*(k) < E^*(1) \quad \text{for every } 1 < k < n.$$

This does not imply that political inequality can occur at literally every possible level of productivity. The two boundary productivity levels are excluded. The strict tournament $k = 1$ produces maximum aggregate output without an effort gap, whereas unrestricted eligibility $k = n$ produces zero aggregate output and no effort gap. Rather, political inequality can arise at every interior productivity level generated by the model.

The second part establishes a stronger separation between productivity and inequality. For any two feasible values of authority capital m and m' ,

$$E^*(k; m) = E^*(k; m')$$

when k is held fixed. However, because individual equilibrium effort levels generally depend on the composition of the subordinate population, it is possible that

$$I^*(k; m) \neq I^*(k; m').$$

Organizations with the same eligibility threshold, and therefore the same aggregate output, may consequently exhibit different effort gaps between loyal and disloyal subordinates.

Finally, the relationship between productivity and political inequality cannot be globally monotonic. Aggregate output is strictly decreasing in the eligibility threshold:

$$E^*(1) > E^*(2) > \dots > E^*(n) = 0.$$

Political inequality, by contrast, is zero at both endpoints but strictly positive under every intermediate rule:

$$I^*(1; m) = I^*(n; m) = 0,$$

and

$$I^*(k; m) > 0 \quad \text{for every } 1 < k < n.$$

Thus, high productivity can coexist with no political inequality under a strict tournament, while lower interior productivity levels can coexist with positive political inequality. At the opposite extreme, the complete collapse of performance incentives produces both zero output and zero effort inequality. Productivity and political inequality therefore need not move together.

Political Inequality in the Final Equilibrium. The results above characterize effort inequality conditional on an eligibility rule k . To characterize political inequality in the final equilibrium, substitute the superior's optimal eligibility rule $k^*(\alpha, m)$ into the effort-gap function:

$$\mathcal{I}^*(\alpha, m) \equiv I^*(k^*(\alpha, m); m). \quad (8)$$

Reliance on loyalty α affects political inequality indirectly through the superior's choice of the eligibility threshold $k^*(\alpha, m)$. Authority capital m , by contrast, affects political inequality through two channels. First, it directly changes the relative effort incentives of loyal and disloyal subordinates, conditional on k . Second, it indirectly affects inequality by changing the superior's optimal institutional choice.

Corollary 2 (Political Inequality in the Final Equilibrium). *Final-equilibrium political inequality is strictly positive if and only if the superior optimally selects an intermediate eligibility rule:*

$$\mathcal{I}^*(\alpha, m) > 0 \quad \Longleftrightarrow \quad 1 < k^*(\alpha, m) < n. \quad (9)$$

Political inequality is therefore endogenous to the superior's competition design. It is not generated by exogenous differences in productive ability or preferences between subordinate types. Instead, it arises because the superior adopts an institutional arrangement in which performance determines eligibility while loyalty determines priority within the eligible set.

Because $k^*(\alpha, m)$ changes discretely across the parameter space, final-equilibrium political inequality need not vary monotonically with either reliance on loyalty or authority capital. Changes in α or m may move the organization into or out of the intermediate institutional region in which performance competition and loyalty-based favoritism operate jointly.

Figure 4 illustrates the relationship among reliance on loyalty α , authority capital m , the superior's optimal eligibility rule $k^*(\alpha, m)$, and final-equilibrium political inequality for $n = 5$ and $V = 1$.

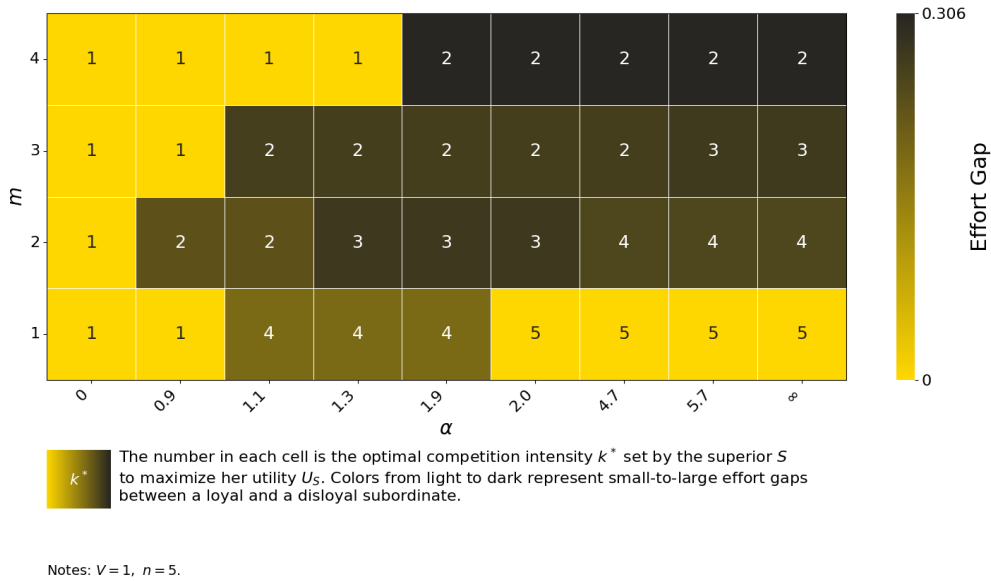


Figure 4 Optimal Eligibility Rules and Politically Induced Effort Inequality

Consistent with Corollary 2, effort gaps appear only in parameter regions in which the superior selects an intermediate eligibility rule. When $k^* = 1$, promotion is determined exclusively by relative performance and loyal and disloyal subordinates exert identical effort. When $k^* = n$, promotion is entirely independent of performance and both types exert zero effort.

The numerical example also illustrates the separation between productivity and political inequality. Aggregate output depends only on the selected eligibility threshold, whereas the effort gap depends additionally on authority capital. Organizations with the same aggregate productivity may therefore exhibit different degrees of politically induced effort inequality.

The figure should be interpreted as an illustration of the general analytical results rather than as their proof. The analytical results apply to any organizational size n , prize value V , and composition m satisfying the assumptions of the model, whereas the figure reports the final-equilibrium outcomes for the specific case $n = 5$ and $V = 1$.

Authority Capital and Institutional Preference

The baseline model takes the legal environment as given and examines how it shapes competition within the organization. We now examine the superior's preference over legal strength and how this preference changes with the accumulation of authority cap-

ital.

Our focus is on institutional incentives rather than realized institutional change. A superior who prefers stronger or weaker legal enforcement may not be able to implement that preference because institutional adjustment can be costly or constrained by other political and institutional forces. Yet the superior's incentive remains a fundamental part of the process. Even when opportunities and institutional capacity for change exist, a superior has little reason to initiate institutional change if doing so does not advance her interests. Understanding when such incentives arise is therefore logically prior to understanding whether they can ultimately be translated into institutional change. Our extension isolates this motivational condition and asks: when a superior can influence legal strength, when does she benefit from stronger legal enforcement, and when does she instead prefer to weaken it?

The central intuition is that the value of legal enforcement depends on the political control the superior already possesses. When her loyal network is small, weakening legal constraints increases her reliance on personal loyalty without ensuring that authority will remain in loyal hands. As the loyal network expands, delegated authority is increasingly likely to remain within that network. Authority capital therefore makes reliance on loyalty less politically risky and weakens the superior's private attachment to strong legal enforcement.

Let α_0 denote the initial legal environment, where a larger α_0 corresponds to greater reliance on loyalty and weaker legal enforcement. Given authority capital m , define the superior's optimized payoff as

$$W(\alpha_0; m) = \max_{k \in \mathcal{K}} \{E^*(k) + \alpha_0 A(k; m)\}. \quad (10)$$

At $\alpha_0 = 0$, the superior values only organizational output. Since $E^*(k)$ is strictly decreasing in k , the strict tournament $k = 1$ is optimal and

$$W(0; m) = E^*(1). \quad (11)$$

We use this payoff as the strong-law benchmark. Because the authority term is measured relative to the impersonal strong-law benchmark (Section), $W(\alpha_0; m)$ and $W(0; m)$ are directly comparable. Define the *institutional tipping point* as

$$\alpha^{TP}(m) = \inf \{\alpha_0 > 0 : W(\alpha_0; m) > W(0; m)\}. \quad (12)$$

The tipping point therefore identifies the boundary beyond which a weaker legal environment yields the superior a higher optimized payoff than the strongest legal environment.

Proposition 6 (Authority Capital and the Institutional Tipping Point). *The institutional tipping point is weakly decreasing in authority capital:*

$$\alpha^{TP}(m+1) \leq \alpha^{TP}(m). \quad (13)$$

Proposition 6 captures the central comparative static. When the superior commands little authority capital, the legal environment must already be relatively weak before further weakening can outperform the strong-law benchmark. As the loyal network expands, this threshold falls. Thus, progressively stronger initial legal environments become vulnerable to institutional weakening.

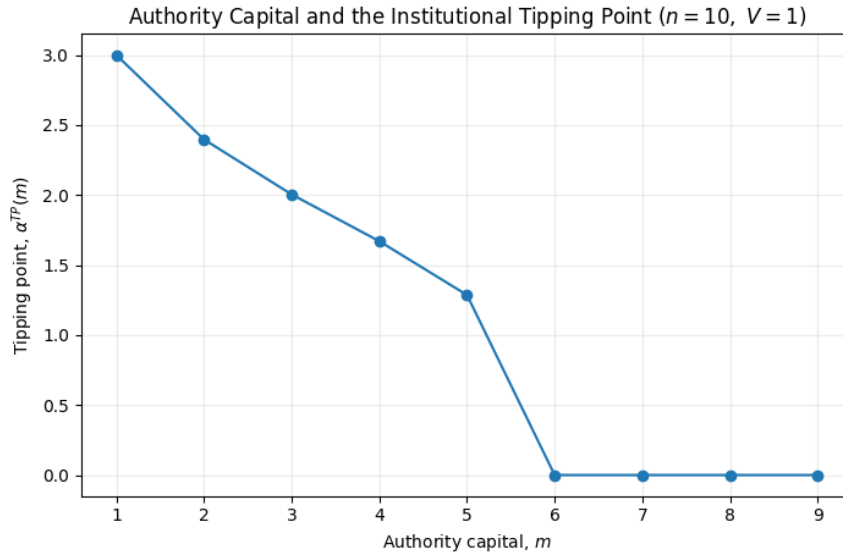


Figure 5 Authority Capital and the Institutional Tipping Point

Notes: The figure plots the institutional tipping point $\alpha^{TP}(m)$ against authority capital m for $n = 10$ and $V = 1$. The tipping point falls as authority capital expands and reaches zero once loyal subordinates constitute a strict majority.

Figure 5 illustrates this relationship.² Authority capital does more than increase the value of loyalty under weak institutions. Its accumulation expands the range of legal environments in which institutional weakening becomes privately attractive.

Institutional Incentive Reversal

The tipping point describes a global comparison with the strong-law benchmark. The superior's incentives before reaching that point reveal an additional feature of the model.

²Figures 5 and 6 use $n = 10$, whereas Figures 1, 3, and 4 use $n = 5$. The smaller organization keeps the earlier figures simple and legible. The larger organization allows authority capital to take more values, which traces out the tipping point and the optimized payoff over a wider range of cases and makes the pattern easier to see. The analytical results hold for every n satisfying the stated assumptions; the choice of n affects only the illustrations.

Because $W(\alpha_0; m)$ is the upper envelope of a finite collection of affine payoff functions, it is continuous, convex, and piecewise linear in α_0 .

When loyal subordinates remain a minority, these properties generate a systematic reversal in the superior's institutional incentives. Let

$$\mathcal{M}(m) = \arg \min_{\alpha_0 \geq 0} W(\alpha_0; m) \quad (14)$$

denote the set of payoff-minimizing legal environments, and define

$$\underline{\alpha}^{\min}(m) = \min \mathcal{M}(m), \quad \bar{\alpha}^{\min}(m) = \max \mathcal{M}(m). \quad (15)$$

Proposition 7 (Institutional Incentive Reversal). *Suppose*

$$m < \frac{n}{2}. \quad (16)$$

Then the superior's optimized payoff initially decreases as legal enforcement weakens, reaches a global minimum, and eventually increases. Moreover,

$$0 < \underline{\alpha}^{\min}(m) \leq \bar{\alpha}^{\min}(m) < \alpha^{TP}(m). \quad (17)$$

Consequently, before the payoff-minimizing region, weakening legal enforcement lowers the superior's payoff. After this region, further weakening raises her payoff, but the strong-law benchmark remains globally superior until α_0 reaches $\alpha^{TP}(m)$. For

$$\alpha_0 > \alpha^{TP}(m), \quad (18)$$

the weaker legal environment yields a strictly higher optimized payoff than the strong-law benchmark.

Proposition 7 distinguishes two different reversals. The payoff minimum marks a *local incentive reversal*: beyond it, further weakening of legal enforcement begins to raise the superior's payoff. The institutional tipping point marks a *global preference reversal*: only after this point does the weaker legal environment outperform the strong-law benchmark. Hence, a superior may begin to benefit from further institutional weakening before weak institutions become globally preferable to strong ones.

Figure 6 illustrates this distinction. With a small loyal network, strong legal enforcement protects the superior from relying on a political resource she does not yet control reliably. As authority capital expands, the recovery in the superior's payoff begins earlier and the global tipping point moves toward stronger legal environments.

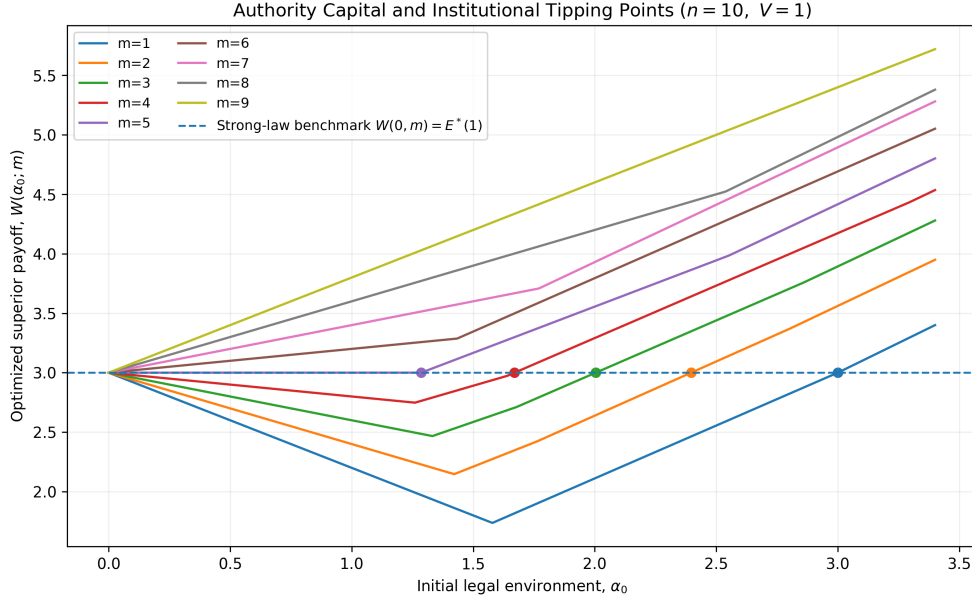


Figure 6 Optimized Payoffs and Institutional Tipping Points

Notes: The figure plots the superior's optimized payoff $W(\alpha_0; m)$ against the initial legal environment α_0 for different levels of authority capital. The horizontal benchmark denotes $W(0; m) = E^*(1)$. For $m < n/2$, the payoff initially declines, reaches a minimum, and subsequently recovers. Its crossing with the strong-law benchmark identifies $\alpha^{TP}(m)$. The figure uses $n = 10$ and $V = 1$.

The Loyal-Majority Threshold

The preceding logic changes qualitatively when loyal subordinates cease to be a minority. Under the strict tournament $k = 1$, the probability of a disloyal winner is

$$P_D^*(1; m) = \frac{n - m}{n}, \quad (19)$$

so the authority value of the strict tournament is

$$A(1; m) = 1 - 2P_D^*(1; m) = \frac{2m}{n} - 1. \quad (20)$$

The sign of this expression changes exactly when loyal subordinates cross one half of the organization.

Proposition 8 (The Loyal-Majority Threshold). *If loyal subordinates constitute a strict majority,*

$$m > \frac{n}{2}, \quad (21)$$

then

$$\alpha^{TP}(m) = 0. \quad (22)$$

Indeed, for every $\alpha_0 > 0$,

$$W(\alpha_0; m) > W(0; m). \quad (23)$$

At the boundary $m = n/2$, the strict tournament has zero authority value, $A(1; m) = 0$, so the superior is locally indifferent to marginal changes in the legal environment while $k = 1$ remains optimal.

The majority threshold has a simple political interpretation. Before a loyal majority emerges, strict performance competition can still place authority disproportionately in disloyal hands. Strong legal enforcement therefore provides the superior with something her loyal network cannot yet reliably supply. Once $m > n/2$, this logic reverses. Even under $k = 1$, where organizational output is maximized, authority is more likely to remain in loyal than disloyal hands. The superior can therefore benefit from greater reliance on loyalty without relaxing performance competition or sacrificing organizational output.

The accumulation of authority capital thus generates three qualitatively distinct institutional environments. With a loyal minority, the superior initially resists institutional weakening. At an evenly divided organization, this initial resistance becomes local neutrality. Once loyal subordinates constitute a majority, even an arbitrarily small movement toward greater reliance on loyalty raises the superior's payoff.

Taken together, the three propositions reveal a feedback between organizational political control and formal institutions. Strong legal institutions shape how superiors use authority capital, but authority capital also changes how superiors value those institutions. As a loyal network expands, the institutional tipping point moves toward stronger legal environments. Once that network becomes a majority, the tipping point disappears altogether.

Implications: Productivity Differences, Political Inequality, and Institutional Stability

Our model has implications that extend beyond the internal design of organizational competition. By linking legal institutions, authority capital, and performance competition within a common framework, the model provides a micro-organizational perspective on several broader questions in political economy. In particular, it offers insights into how institutional environments shape productivity and its stability across political turnover, how political inequality can emerge within formally common competitive rules, and how institutional design can contribute to the stability of formal institutions.

First, the model provides a mechanism linking institutional environments to differences in organizational performance. Legal strength affects productivity not directly, but by changing the superior's reliance on loyalty and, consequently, the intensity of performance competition she is willing to sustain. This mechanism also generates an implication for political turnover. Under strong legal enforcement, changes in author-

ity capital have little effect on the optimal competition rule, insulating organizational performance from changes in political leadership. Under weak legal enforcement, by contrast, performance becomes more dependent on the authority capital of the incumbent. An incoming superior with a limited loyal network may initially restrict performance competition to increase the likelihood that authority is allocated to loyal subordinates. As authority capital accumulates, political control can increasingly be secured through the loyal network itself, allowing performance competition to intensify. The model therefore suggests that strong legal institutions may contribute not only to more intensive performance competition and higher organizational output, but also to greater performance stability across political turnover, whereas performance under weak legal institutions may be more sensitive to changes in political leadership.

Second, the model also offers a different perspective on political inequality. Inequality is often understood in terms of differences in outcomes, such as income, office, or access to resources. Our model shows that political inequality can also arise through differences in incentives within a common competitive process. When promotion depends partly on political loyalty, loyal and disloyal subordinates face different returns to effort and therefore exert different levels of effort under the same formal promotion rule. Importantly, this effort gap is nonmonotonic in the intensity of performance competition. It disappears both when competition is sufficiently strict to leave no scope for promotion discretion and when performance competition is sufficiently weak that neither group exerts effort. Thus, the absence of an observed effort gap does not necessarily imply the absence of political differentiation: it can arise either because promotion is determined by performance or because performance no longer meaningfully affects advancement.

Third, the model extension also offers an implication for institutional stability. Strong legal institutions need not remain stable simply because they are initially strong. As authority capital accumulates, the superior's institutional tipping point falls, making progressively stronger legal environments vulnerable to weakening. This suggests that institutional resilience depends not only on protecting formal legal constraints, but also on limiting the accumulation of loyalty-based political control within organizations. Institutional arrangements that restrict the superior's ability to convert appointment and promotion authority into a personal network of loyal subordinates can therefore serve a second function: they preserve the superior's own incentive to maintain strong legal institutions. In this sense, safeguards against institutional erosion may operate not only by making formal constraints harder to dismantle, but also by preventing the accumulation of the organizational resources that make dismantling them privately attractive.

Conclusion

A recurring concern in political science is how compliance and order are sustained in organizations, including bureaucracies, parties, firms, and states, when formal legal enforcement is weak or uncertain. Studies of clientelism, patronage, and informal institutions have long shown that loyalty-based relationships can substitute for formal rules. What remains less understood is how this substitution affects the internal organization of competition: when reliance on loyalty changes the superior's willingness to sustain performance competition, and what this implies for organizational performance and political inequality.

This paper develops a theory of this trade-off. When legal enforcement is strong, the superior can rely more heavily on formal institutions and has less reason to sacrifice performance competition for political control. As legal enforcement weakens, personal loyalty becomes more valuable, increasing the superior's incentive to retain discretion over advancement. Authority capital, captured by the superior's stock of loyal subordinates, changes this trade-off by affecting how readily political control can be secured through the composition of the organization itself. The interaction between legal strength and authority capital therefore determines how political control is organized and how much performance competition the superior is willing to sustain.

These organizational choices have consequences beyond the allocation of authority itself. They shape aggregate effort and political inequality and determine how sensitive organizational performance is to changes in political leadership. The analysis also reveals a feedback from organizational political control to institutional incentives. As authority capital accumulates, the institutional tipping point at which weaker legal enforcement becomes privately beneficial falls. Formal institutions therefore shape the organizational value of loyalty, while the accumulation of authority capital can in turn alter the incentives to preserve those institutions.

More broadly, the analysis suggests that authority capital should be treated as an important organizational variable in the study of political and economic performance. Formal institutions alone do not determine how authority is exercised within organizations, just as the internal organization of political control need not be neutral for the stability of formal institutions. Understanding productivity, political inequality, and institutional stability therefore requires attention to both the formal rules governing political and economic interaction and the organizational distribution of authority that develops within them.

Appendix

This appendix derives every result whose proof the main text defers here. Appendix derives the contest success function used throughout. Appendix proves the results stated in Section . Appendix proves the results of Section .

The Berry Multi-Winner Contest Success Function

The winning probabilities used in the main text are an application of Berry's (1993) multi-winner contest success function, which this section derives from the single-winner Tullock contest success function (TSF) it generalizes.

Let $x = (x_1, \dots, x_n)$ denote a vector of efforts for n players. A single-winner contest success function assigns each player i a winning probability $Prob^i(x)$ satisfying three standard properties (Skaperdas 1996): (1A) probabilities are nonnegative and sum to one, and $Prob^i(x) > 0$ whenever $x_i > 0$; (2A) $Prob^i(x)$ is increasing in x_i and decreasing in every $x_j, j \neq i$; (3A) $Prob^i(x)$ is invariant to relabeling players. The Tullock success function,

$$Prob^i(x) = \frac{x_i^r}{x_1^r + x_2^r + \dots + x_n^r}, \quad (A1)$$

satisfies all three properties, where r indexes the responsiveness of winning probability to effort (Rai and Sarin 2009).

Berry (1993) extends (A1) to a k -winner contest ($1 < k \leq n$) by summing, over every combination in which player i is one of the k winners, the TSF-weighted share of that outcome. The numerator sums x_i^r across the $\binom{n-1}{k-1}$ combinations of the remaining $k-1$ winners together with the efforts of those co-winners; the denominator sums the same TSF-style expression across all $\binom{n}{k}$ possible winning sets. The resulting probability that player i is among the top k is

$$Prob_k^i(x) = \frac{\binom{n-1}{k-1} x_i^r + \binom{n-2}{k-2} \sum_{j \neq i} x_j^r}{\binom{n-1}{k-1} \sum_{z=1}^n x_z^r}, \quad j, i \in N. \quad (A2)$$

Setting $r = 1$ in (A2) and applying the stochastic-favoritism selection rule of Section (a loyal winner is drawn uniformly from the loyal members of the top- k set whenever one exists, and a disloyal winner only otherwise) yields the individual winning probabilities P_{li}, P_{dj} used throughout the main text and this appendix.

Setting $r = 1$, the probability that a particular k -subset S of subordinates forms the eligible set is

$$\Pr(S \mid x) = \frac{\sum_{j \in S} x_j}{\binom{n-1}{k-1} \sum_{z=1}^n x_z}, \quad (A3)$$

which sums to one over all k -subsets because each x_j appears in exactly $\binom{n-1}{k-1}$ of them.

When every subordinate exerts zero effort, we adopt the standard convention that the eligible set is drawn uniformly from all $\binom{n}{k}$ subsets.

General Proofs for the Baseline Model

Throughout this section let

$$n_d \equiv n - m$$

denote the number of disloyal subordinates, and let $\mathcal{K} \equiv \{1, \dots, n\}$ be the superior's feasible set of competition rules. Including $k = n$ is necessary because $k = n$ is the case in which eligibility, and hence promotion, is completely independent of performance. Unless otherwise stated, results comparing loyal and disloyal agents presume $1 \leq m \leq n - 1$; the boundary cases $m = 0$ and $m = n$ contain only one type.

Anticipating the main text's Proposition 1, define

$$E^*(k) \equiv \sqrt{\frac{V(n-k)}{k}}, \quad k \in \mathcal{K}, \quad (\text{A4})$$

with $E^*(n) = 0$; this section derives, rather than assumes, that equilibrium aggregate effort equals this expression. For $k \leq n_d$, also define

$$R_k(m) \equiv \frac{\binom{n-1}{k}}{\binom{n_d}{k}}, \quad Q_k(m) \equiv k + n_d R_k(m), \quad (\text{A5})$$

and

$$B_k(m) \equiv \frac{n_d R_k(m) - n_d + k}{m}. \quad (\text{A6})$$

$B_k(m)$ is the quantity denoted $B(k, m, n)$ in the main text (eq. (3)), with the dependence on n suppressed since $n_d \equiv n - m$.

Preliminary binomial identities

We repeatedly use Vandermonde's identity,

$$\sum_r \binom{a}{r} \binom{b}{s-r} = \binom{a+b}{s}, \quad (\text{A7})$$

where a binomial coefficient is understood to be zero whenever its lower index lies outside its admissible range. We also use

$$\binom{a}{b-1} \frac{a-b+1}{b} = \binom{a}{b} \quad (\text{A8})$$

and

$$n_d \binom{n_d - 1}{k - 1} = k \binom{n_d}{k}. \quad (\text{A9})$$

The symmetric effort equilibrium

For any subordinate i and any k -subset S , let $w_i(S) \in [0, 1]$ denote the probability that i receives the prize conditional on S being the eligible set. Under stochastic favoritism, $w_i(S)$ depends only on the types of the members of S , not on effort, and $w_i(S) = 0$ whenever $i \notin S$.

Lemma A.1 (Concavity of individual payoffs). *Fix $k < n$, a subordinate i , and the efforts x_{-i} of all other subordinates, and suppose $c_i \equiv \sum_{j \neq i} x_j > 0$. Then i 's winning probability can be written as*

$$P_i(x_i; x_{-i}) = \frac{a_i x_i + b_i}{\binom{n-1}{k-1} (x_i + c_i)}, \quad a_i \equiv \sum_{S \ni i} w_i(S), \quad b_i \equiv \sum_{S \ni i} w_i(S) \sum_{j \in S \setminus \{i\}} x_j, \quad (\text{A10})$$

and satisfies

$$a_i c_i - b_i = \sum_{S \ni i} w_i(S) \sum_{j \notin S} x_j \geq 0. \quad (\text{A11})$$

Consequently, P_i is nondecreasing and concave in x_i on $[0, \infty)$, the payoff $\pi_i(x_i) \equiv P_i(x_i; x_{-i})V - x_i^2/2$ is strictly concave in x_i , and i has a unique best response: $x_i = 0$ if $V \partial P_i / \partial x_i|_{x_i=0} \leq 0$, and otherwise the unique positive solution of the first-order condition $x_i = V \partial P_i / \partial x_i$.

Proof. By (A3),

$$P_i(x_i; x_{-i}) = \sum_S \Pr(S | x) w_i(S) = \frac{\sum_{S \ni i} w_i(S) (x_i + \sum_{j \in S \setminus \{i\}} x_j)}{\binom{n-1}{k-1} (x_i + c_i)},$$

which is (A10). For each $S \ni i$, $c_i = \sum_{j \in S \setminus \{i\}} x_j + \sum_{j \notin S} x_j$, so multiplying by $w_i(S)$ and summing over $S \ni i$ gives (A11); nonnegativity follows because every term is nonnegative. Differentiating (A10),

$$\frac{\partial P_i}{\partial x_i} = \frac{a_i c_i - b_i}{\binom{n-1}{k-1} (x_i + c_i)^2} \geq 0, \quad \frac{\partial^2 P_i}{\partial x_i^2} = -\frac{2(a_i c_i - b_i)}{\binom{n-1}{k-1} (x_i + c_i)^3} \leq 0.$$

Hence P_i is nondecreasing and concave in x_i . Since $-x_i^2/2$ is strictly concave, π_i is strictly concave on $[0, \infty)$, and $\pi_i(x_i) \rightarrow -\infty$ as $x_i \rightarrow \infty$ because $P_i \leq 1$. A strictly concave function on $[0, \infty)$ that diverges to $-\infty$ has a unique maximizer, characterized by the Kuhn–Tucker conditions stated in the lemma. $\square$

Lemma A.2 (Symmetric equilibrium effort). *Suppose $1 \leq m \leq n - 1$ and $V > 0$. For every $k \in \mathcal{K}$, the subordinate effort subgame has a unique type-symmetric Nash equilibrium, given as follows.*

1. *If $k \leq n_d$, the equilibrium is*

$$x_d^*(k; m) = \frac{E^*(k)}{Q_k(m)}, \quad x_l^*(k; m) = B_k(m) \frac{E^*(k)}{Q_k(m)}. \quad (\text{A12})$$

2. *If $n_d < k < n$, the equilibrium is*

$$x_d^*(k; m) = 0, \quad x_l^*(k; m) = \frac{E^*(k)}{m}. \quad (\text{A13})$$

3. *If $k = n$, the equilibrium is*

$$x_d^*(n; m) = x_l^*(n; m) = 0. \quad (\text{A14})$$

Proof. We first consider $k \leq n_d$. At a type-symmetric effort profile, write the effort of every loyal agent as x_l and that of every disloyal agent as x_d . Differentiating the contest-success functions with respect to an individual agent's own effort and then imposing type symmetry gives

$$\frac{\partial P_{li}}{\partial x_{li}} = \frac{x_l \sum_{q=1}^{\min\{m,k\}} \binom{m-1}{q} \binom{n_d}{k-q} + \frac{n_d}{m} x_d \sum_{q=1}^{\min\{m,k\}} \binom{m}{q} \binom{n_d-1}{k-q}}{\binom{n-1}{k-1} (mx_l + n_d x_d)^2}, \quad (\text{A15})$$

$$\frac{\partial P_{dj}}{\partial x_{dj}} = \frac{\frac{m}{n_d} \binom{n_d}{k} x_l + \binom{n_d-1}{k} x_d}{\binom{n-1}{k-1} (mx_l + n_d x_d)^2}. \quad (\text{A16})$$

By Vandermonde's identity,

$$\sum_{q=1}^{\min\{m,k\}} \binom{m-1}{q} \binom{n_d}{k-q} = \binom{n-1}{k} - \binom{n_d}{k}, \quad (\text{A17})$$

$$\sum_{q=1}^{\min\{m,k\}} \binom{m}{q} \binom{n_d-1}{k-q} = \binom{n-1}{k} - \binom{n_d-1}{k}. \quad (\text{A18})$$

The first-order conditions are

$$x_l = V \frac{\partial P_{li}}{\partial x_{li}}, \quad x_d = V \frac{\partial P_{dj}}{\partial x_{dj}}. \quad (\text{A19})$$

Because all coefficients in (A15)–(A16) are nonnegative and at least one coefficient in each expression is positive, an equilibrium in this region has $x_l > 0$ and $x_d > 0$. Let

$B = x_l/x_d$. Dividing the loyal first-order condition by the disloyal first-order condition, and using (A17)–(A18), yields the quadratic condition

$$B \left[\frac{m}{n_d} \binom{n_d}{k} B + \binom{n_d-1}{k} \right] = B \left[\binom{n-1}{k} - \binom{n_d}{k} \right] + \frac{n_d}{m} \left[\binom{n-1}{k} - \binom{n_d-1}{k} \right]. \quad (\text{A20})$$

The left side minus the right side is a strictly convex quadratic in B with a negative constant term. It therefore has exactly one positive root. Direct substitution, using $\binom{n_d-1}{k} = \frac{n_d-k}{n_d} \binom{n_d}{k}$, shows that this root is

$$B = B_k(m) = \frac{n_d \binom{n-1}{k} / \binom{n_d}{k} - n_d + k}{m}. \quad (\text{A21})$$

Thus (A6) is algebraically equivalent to the positive-root formula obtained from (A20), but is considerably simpler.

Substituting $x_l = B_k x_d$ into the disloyal first-order condition gives

$$(x_d^*)^2 = \frac{V \binom{n_d}{k} (m B_k + n_d - k)}{\binom{n-1}{k-1} n_d (m B_k + n_d)^2}. \quad (\text{A22})$$

From (A6),

$$m B_k + n_d = Q_k \quad \text{and} \quad m B_k + n_d - k = n_d R_k. \quad (\text{A23})$$

Moreover, (A8) implies

$$\frac{\binom{n_d}{k} n_d R_k}{\binom{n-1}{k-1} n_d} = \frac{\binom{n-1}{k}}{\binom{n-1}{k-1}} = \frac{n-k}{k}. \quad (\text{A24})$$

Equations (A22)–(A24) therefore give

$$x_d^* = \frac{E^*(k)}{Q_k}, \quad x_l^* = B_k \frac{E^*(k)}{Q_k}, \quad (\text{A25})$$

which proves part 1.

Now suppose $n_d < k < n$. Every top- k set necessarily contains at least one loyal agent. A disloyal agent consequently has zero probability of receiving the prize for every effort choice. Since effort is costly, $x_d^* = 0$. Conditional on type symmetry among loyal agents, differentiation of a loyal agent's winning probability gives

$$\frac{\partial P_{li}}{\partial x_{li}} = \frac{\sum_{j=0}^{n_d} \binom{n_d}{j} \binom{m-1}{k-j}}{\binom{n-1}{k-1} m^2 x_l}. \quad (\text{A26})$$

Vandermonde's identity reduces the sum in the numerator to $\binom{n-1}{k}$. Hence the loyal

first-order condition is

$$x_l^2 = \frac{V}{m^2} \frac{\binom{n-1}{k}}{\binom{n-1}{k-1}} = \frac{V(n-k)}{m^2 k}, \quad (\text{A27})$$

which establishes (A13).

Finally, if $k = n$, every subordinate is eligible independently of performance. Winning probabilities are therefore locally and globally invariant to effort. The unique cost-minimizing choice of each subordinate is zero, proving part 3.

It remains to verify that the profiles above are equilibria and that no other type-symmetric profile is. We first rule out the zero-effort profile when $k < n$. At that profile, by the convention following (A3), a loyal subordinate i wins with probability $p_0 \equiv \sum_{S \ni i} w_i(S) / \binom{n}{k} > 0$. If i deviates to any $\varepsilon > 0$, (A3) implies that the eligible set is drawn uniformly from the subsets containing i , so i wins with probability $\sum_{S \ni i} w_i(S) / \binom{n-1}{k-1} = (n/k) p_0 > p_0$. The gain $V(n/k - 1)p_0$ is independent of ε , while the cost $\varepsilon^2/2$ vanishes as $\varepsilon \rightarrow 0$, so a sufficiently small deviation is profitable. Hence the zero-effort profile is not an equilibrium for any $k < n$.

Case $k \leq n_d$. Consider a type-symmetric profile in which exactly one type exerts zero effort. If $x_d = 0 < x_l$, take any disloyal i and any all-disloyal set $S \ni i$, which exists because $k \leq n_d$; then $w_i(S) = 1/k > 0$ and every loyal subordinate lies outside S with positive effort, so (A11) is strictly positive and i 's marginal return at zero effort is positive. If $x_l = 0 < x_d$, take any loyal i and any set $S \ni i$ that excludes some disloyal subordinate, which exists because $k < n$; then $w_i(S) > 0$ and that disloyal subordinate contributes positive effort outside S , so again (A11) is strictly positive. In either case the zero-effort type has a profitable deviation, so every type-symmetric equilibrium is interior. At an interior profile, the first-order conditions (A19) have exactly one solution, because (A20) has exactly one positive root and (A22) then determines $x_d^* > 0$ uniquely. Since $c_i > 0$ for every subordinate at this solution, Lemma A.1 implies that the first-order conditions are sufficient, so (A12) is an equilibrium and the unique type-symmetric one.

Case $n_d < k < n$. Every eligible set contains a loyal subordinate, so $w_i(S) = 0$ for every disloyal i and every S . A disloyal subordinate's winning probability is therefore identically zero, and $x_d = 0$ is her unique best response. Because $k < n$, the loyal type cannot also exert zero effort, since the zero-effort profile is not an equilibrium. Moreover, $m = n - n_d > n - k \geq 1$, so $m \geq 2$ and $c_i = (m - 1)x_l > 0$ for every loyal i whenever $x_l > 0$. The loyal first-order condition has the unique positive solution (A13), and Lemma A.1 implies that it is a best response. Hence (A13) is the unique type-symmetric equilibrium.

Case $k = n$. Winning probabilities do not depend on effort, so zero effort is every subordinate's unique best response, and (A14) is the unique equilibrium, type-symmetric or not. $\square$

Remark A.1 (Boundary populations). *If $m = 0$, all agents are disloyal; if $m = n$, all are loyal. In either case the standard symmetric multiple-winner contest gives individual effort $E^*(k)/n$ for $k < n$, aggregate effort $E^*(k)$, and zero effort for $k = n$. Expressions involving the ratio $B_k = x_l/x_d$ are, of course, inapplicable when one type is absent.*

Effort asymmetry and effort collapse

Lemma A.3 (Effort asymmetry under stochastic favoritism). *Suppose $1 \leq m \leq n - 1$. In the symmetric equilibrium,*

$$x_l^*(k; m) \geq x_d^*(k; m). \quad (\text{A28})$$

Equality holds at $k = 1$ and $k = n$. For every intermediate rule $1 < k < n$, the inequality is strict.

Proof. First suppose $k \leq n_d$. From Lemma A.2, $x_l^*/x_d^* = B_k$. At $k = 1$,

$$R_1 = \frac{n-1}{n_d}, \quad B_1 = \frac{n_d(n-1)/n_d - n_d + 1}{m} = 1. \quad (\text{A29})$$

For $k > 1$, use Vandermonde's identity to write

$$\binom{n-1}{k} = \sum_{r=0}^{\min\{m-1, k\}} \binom{m-1}{r} \binom{n_d}{k-r}. \quad (\text{A30})$$

The $r = 0$ and $r = 1$ terms alone imply

$$R_k \geq 1 + (m-1) \frac{\binom{n_d}{k-1}}{\binom{n_d}{k}} = 1 + (m-1) \frac{k}{n_d - k + 1}. \quad (\text{A31})$$

Substituting this bound into (A6) gives $B_k > 1$ for $1 < k \leq n_d$. Hence $x_l^* > x_d^* > 0$ in this region.

If $n_d < k < n$, Lemma A.2 gives $x_l^* = E^*(k)/m > 0 = x_d^*$. If $k = n$, both efforts equal zero. Combining the three cases proves the result. $\square$

Corollary A.1 (Effort collapse under degenerate competition). *Suppose $n_d < k$. If $k < n$, disloyal agents exert zero effort while loyal agents exert $E^*(k)/m > 0$. If $k = n$, both types exert zero effort.*

Proof. Immediate from parts 2 and 3 of Lemma A.2. $\square$

Aggregate effort

Proposition A.1 (Aggregate-output invariance). *For every $n \geq 2$, $V > 0$, $m \in \{0, \dots, n\}$, and $k \in \mathcal{K}$, aggregate equilibrium effort satisfies*

$$mx_l^*(k; m) + n_d x_d^*(k; m) = E^*(k) = \sqrt{\frac{V(n-k)}{k}}. \quad (\text{A32})$$

Consequently, aggregate effort is independent of m conditional on k and is strictly decreasing in k for $k < n$.

Proof. If $k \leq n_d$, equations (A12) and (A23) imply

$$mx_l^*(k; m) + n_d x_d^*(k; m) = (mB_k + n_d) \frac{E^*(k)}{Q_k} = E^*(k). \quad (\text{A33})$$

If $n_d < k < n$, equation (A13) gives

$$mx_l^*(k; m) + n_d x_d^*(k; m) = m \frac{E^*(k)}{m} + n_d \cdot 0 = E^*(k). \quad (\text{A34})$$

At $k = n$, both sides of (A32) equal zero. The same formula applies in the one-type boundary populations described above.

Independence from m follows directly from (A32). To establish monotonicity in k , observe that

$$E^*(k)^2 = V \left(\frac{n}{k} - 1 \right), \quad (\text{A35})$$

which is strictly decreasing in k . Since $E^*(k) \geq 0$, $E^*(k)$ itself is strictly decreasing on $\{1, \dots, n\}$. $\square$

The probability of a disloyal winner

Lemma A.4 (Equilibrium probability of a disloyal winner). *Suppose $1 \leq m \leq n - 1$. In equilibrium,*

$$P_D^*(k; m) = \begin{cases} \frac{n-k}{R_k(m) [k + n_d R_k(m)]}, & k \leq n_d, \\ 0, & k > n_d. \end{cases} \quad (\text{A36})$$

Moreover, $P_D^*(k; m)$ is strictly decreasing in k on $\{1, \dots, n_d + 1\}$ and remains zero thereafter.

Proof. For $k \leq n_d$, the aggregate probability that a disloyal agent wins is

$$P_D^*(k; m) = \frac{n_d \binom{n_d-1}{k-1} x_d^*}{\binom{n-1}{k-1} (n_d x_d^* + m x_l^*)}. \quad (\text{A37})$$

Using (A9), $x_d^*/E^*(k) = 1/Q_k$, and $\binom{n-1}{k-1} = \frac{k}{n-k} \binom{n-1}{k}$, equation (A37) becomes

$$\begin{aligned} P_D^*(k; m) &= \frac{k \binom{n_d}{k}}{\binom{n-1}{k-1} Q_k} \\ &= \frac{n-k}{R_k(k + n_d R_k)}, \end{aligned} \quad (\text{A38})$$

which is (A36). If $k > n_d$, every eligible set contains a loyal agent, so stochastic favoritism makes the probability of a disloyal winner zero.

It remains to prove monotonicity. For $k < n_d$,

$$\frac{R_{k+1}}{R_k} = \frac{n-1-k}{n_d-k} = 1 + \frac{m-1}{n_d-k} \geq 1. \quad (\text{A39})$$

Thus, as k rises, the numerator $n-k$ in (A36) strictly decreases, while both factors in its denominator weakly increase and the factor $k + n_d R_k$ strictly increases. Therefore $P_D^*(k+1; m) < P_D^*(k; m)$ for $k < n_d$. Finally, $P_D^*(n_d; m) > 0 = P_D^*(n_d+1; m)$, completing the proof. $\square$

Two useful boundary values follow immediately:

$$P_D^*(1; m) = \frac{n_d}{n} = \frac{n-m}{n}, \quad P_D^*(k; m) = 0 \quad \text{for all } k > n_d. \quad (\text{A40})$$

The superior's optimal competition rule

Substituting equilibrium effort into the superior's payoff gives the reduced problem

$$U_S(k; \alpha, m) = E^*(k) + \alpha A(k; m), \quad A(k; m) \equiv 1 - 2P_D^*(k; m), \quad (\text{A41})$$

which is the closed-form counterpart of the main text's eq. (1), with $A(k; m)$ the same expected-authority-value quantity defined there. Lemma A.4 implies that $A(k; m)$ is weakly increasing in k and strictly increasing until it reaches its upper bound of one.

Because the maximizer need not be unique at switching values, define the optimal correspondence

$$K^*(\alpha, m) \equiv \arg \max_{k \in \mathcal{K}} U_S(k; \alpha, m), \quad (\text{A42})$$

and its smallest and largest selections by

$$\underline{k}^*(\alpha, m) = \min K^*(\alpha, m), \quad \bar{k}^*(\alpha, m) = \max K^*(\alpha, m). \quad (\text{A43})$$

Lemma A.5 (Benchmark without reliance on loyalty). *If $\alpha = 0$, the superior uniquely chooses $k^* = 1$.*

Proof. At $\alpha = 0$, equation (A41) reduces to $U_S(k; 0, m) = E^*(k)$. Proposition A.1 estab-

lishes that $E^*(k)$ is strictly decreasing in k . Hence its unique maximizer is $k = 1$. $\square$

Proposition A.2 (Optimal competition and legal weakness). *For fixed n , V , and m , both $\underline{k}^*(\alpha, m)$ and $\bar{k}^*(\alpha, m)$ are weakly increasing in α . Equivalently, weaker legal enforcement, represented by a larger α , cannot induce the superior to choose a strictly more intense competition rule.*

Proof. Let $k' > k$ and $\alpha' > \alpha$. From (A41),

$$\begin{aligned} & [U_S(k'; \alpha', m) - U_S(k; \alpha', m)] - [U_S(k'; \alpha, m) - U_S(k; \alpha, m)] \\ & = (\alpha' - \alpha)[A(k'; m) - A(k; m)] \geq 0, \end{aligned} \quad (\text{A44})$$

where the inequality follows from Lemma A.4. Thus U_S has increasing differences in (k, α) . Since $\mathcal{K}$ is a finite chain, Topkis's monotonicity theorem implies that $K^*(\alpha, m)$ is increasing in α in the strong set order, so both its smallest and its largest elements are weakly increasing in α . $\square$

Remark A.2 (Global switching conditions). *Adjacent comparisons alone do not generally characterize the global optimum. For any $\ell > k$ with $A(\ell; m) > A(k; m)$, define the pairwise cutoff*

$$\hat{\alpha}_{k,\ell}(m) \equiv \frac{E^*(k) - E^*(\ell)}{A(\ell; m) - A(k; m)} > 0. \quad (\text{A45})$$

Then ℓ weakly dominates k if and only if $\alpha \geq \hat{\alpha}_{k,\ell}(m)$. A rule k for which no smaller rule has the same value of A is globally optimal if and only if it beats every alternative:

$$\max_{j < k: A(k; m) > A(j; m)} \hat{\alpha}_{j,k}(m) \leq \alpha \leq \min_{\ell > k: A(\ell; m) > A(k; m)} \hat{\alpha}_{k,\ell}(m), \quad (\text{A46})$$

with the usual conventions for an empty maximum or minimum. If the interval in (A46) is empty, k is never optimal. This formulation allows the optimal rule to jump over one or more values of k . It does not require the adjacent cutoffs to be ordered. If some $j < k$ satisfies $A(j; m) = A(k; m)$, then $E^*(j) > E^*(k)$ and j strictly dominates k for every α ; such a k is never optimal.

Corollary A.2 (Induced productivity effect of legal weakness). *For fixed n , V , and m , equilibrium aggregate output under any monotone selection of the optimal competition correspondence is weakly decreasing in α .*

Proof. Proposition A.2 establishes that the selected k^* is weakly increasing in α , while Proposition A.1 establishes that $E^*(k)$ is strictly decreasing in k . Their composition $E^*(k^*(\alpha, m))$ is therefore weakly decreasing in α . $\square$

Effort gaps

Proposition A.3 (Political origin and incidence of effort gaps). *Suppose both types are present, $1 \leq m \leq n-1$. Conditional on a competition rule k , the equilibrium effort gap is*

$$I^*(k; m) \equiv x_l^*(k; m) - x_d^*(k; m) = \begin{cases} \frac{[B_k(m) - 1]E^*(k)}{Q_k(m)}, & k \leq n_d, \\ \frac{E^*(k)}{m}, & n_d < k < n, \\ 0, & k = n. \end{cases} \quad (\text{A47})$$

Consequently,

$$I^*(k; m) \begin{cases} = 0, & k \in \{1, n\}, \\ > 0, & 1 < k < n. \end{cases} \quad (\text{A48})$$

Thus effort inequality arises exactly when the competition rule leaves room for loyalty-based priority while retaining some performance dependence.

Proof. Equation (A47) follows by subtracting the two effort expressions in Lemma A.2. Lemma A.3 establishes that $B_1 = 1$, $B_k > 1$ for $1 < k \leq n_d$, and that loyal effort is strictly positive while disloyal effort is zero for $n_d < k < n$. At $k = n$, both types exert zero effort. These observations give (A48). $\square$

Remark A.3 (Shape of the effort gap). *Equation (A48) establishes that the effort gap vanishes at both extremes and is strictly positive at every intermediate rule. It does not establish that the sequence $I^*(1; m), \dots, I^*(n; m)$ is single-peaked, and we make no such claim; single-peakedness would require a direct comparison of $I^*(k+1; m)$ and $I^*(k; m)$ using (A47).*

Political inequality in the final equilibrium

Corollary A.3 (Political Inequality in the Final Equilibrium). $\mathcal{I}^*(\alpha, m) > 0$ if and only if $1 < k^*(\alpha, m) < n$.

Proof. By Lemma A.3 (equivalently, eq. (A48)), $I^*(k; m) \equiv x_l^*(k; m) - x_d^*(k; m) = 0$ exactly when $k \in \{1, n\}$, and $I^*(k; m) > 0$ for every $1 < k < n$. Since $k^*(\alpha, m) \in \{1, \dots, n\}$ by construction, substituting $k = k^*(\alpha, m)$ gives $\mathcal{I}^*(\alpha, m) \equiv I^*(k^*(\alpha, m); m) > 0$ if and only if $k^*(\alpha, m) \notin \{1, n\}$, i.e., $1 < k^*(\alpha, m) < n$. $\square$

Authority capital and the optimal competition rule

Proposition 3 describes three regimes through which authority capital m affects the optimal eligibility rule: neutrality, discretion activation, and authority–discretion substitution. We first establish a general sufficient condition for substitution that holds for every $\alpha \geq 0$ (Proposition A.4) and show that it reduces to a comparison of adjacent rules (Lemma A.6). We then show that the two extremes of reliance on loyalty admit sharper characterizations, valid for every n and V (Propositions A.5 and A.6). Finally, a counterexample shows that the substitution condition can fail at intermediate reliance on loyalty, which is where discretion activation arises (Proposition A.7).

Proposition A.4 (Authority–discretion substitution). *Fix $n, V, \alpha \geq 0$, and $m \in \{0, \dots, n-1\}$. Suppose*

$$G_{k,k'}(m+1) \leq G_{k,k'}(m) \quad \text{for every } k, k' \in \mathcal{K} \text{ with } k < k'. \quad (\text{A49})$$

Then, for every $k \in K^*(\alpha, m)$ and $k' \in K^*(\alpha, m+1)$,

$$\min\{k, k'\} \in K^*(\alpha, m+1) \quad \text{and} \quad \max\{k, k'\} \in K^*(\alpha, m). \quad (\text{A50})$$

Consequently,

$$\underline{k}^*(\alpha, m+1) \leq \underline{k}^*(\alpha, m) \quad \text{and} \quad \bar{k}^*(\alpha, m+1) \leq \bar{k}^*(\alpha, m), \quad (\text{A51})$$

and, whenever the optimal rule is unique at both m and $m+1$, $k^*(\alpha, m+1) \leq k^*(\alpha, m)$. Accordingly, equilibrium output evaluated at either extremal selection weakly increases from m to $m+1$.

Proof. For any $k < k'$, the payoff difference between the broader and the stricter rule is

$$\Delta U_{k,k'}(m) \equiv U_S(k'; \alpha, m) - U_S(k; \alpha, m) = [E^*(k') - E^*(k)] + \alpha G_{k,k'}(m),$$

where the output term does not depend on m by Proposition A.1. Hence

$$\Delta U_{k,k'}(m+1) - \Delta U_{k,k'}(m) = \alpha [G_{k,k'}(m+1) - G_{k,k'}(m)] \leq 0, \quad (\text{A52})$$

by (A49) and $\alpha \geq 0$. That is, U_S has decreasing differences in (k, m) .

Let $k \in K^*(\alpha, m)$ and $k' \in K^*(\alpha, m+1)$. If $k' \leq k$, then (A50) holds trivially. Suppose instead that $k' > k$.

First, optimality of k at m gives $\Delta U_{k,k'}(m) \leq 0$. By (A52), $\Delta U_{k,k'}(m+1) \leq \Delta U_{k,k'}(m) \leq 0$, so $U_S(k; \alpha, m+1) \geq U_S(k'; \alpha, m+1)$. Since k' attains the maximum at $m+1$, so does k . Hence $k = \min\{k, k'\} \in K^*(\alpha, m+1)$.

Second, optimality of k' at $m+1$ gives $\Delta U_{k,k'}(m+1) \geq 0$. By (A52), $\Delta U_{k,k'}(m) \geq$

$\Delta U_{k,k'}(m+1) \geq 0$, so $U_S(k'; \alpha, m) \geq U_S(k; \alpha, m)$. Since k attains the maximum at m , so does k' . Hence $k' = \max\{k, k'\} \in K^*(\alpha, m)$. This proves (A50).

For the smallest selections, let $k = \underline{k}^*(\alpha, m)$ and $k' = \underline{k}^*(\alpha, m+1)$. If $k' > k$, then (A50) implies $k \in K^*(\alpha, m+1)$ with $k < k'$, contradicting the minimality of k' . Hence $k' \leq k$.

For the largest selections, let $k = \bar{k}^*(\alpha, m)$ and $k' = \bar{k}^*(\alpha, m+1)$. If $k' > k$, then (A50) implies $k' \in K^*(\alpha, m)$ with $k' > k$, contradicting the maximality of k . Hence $k' \leq k$.

The statement for a unique optimum follows because the smallest and largest selections then coincide with k^* . The output claim follows from the strict monotonicity of E^* in k (Proposition A.1), which gives $\hat{E}(\alpha, m+1) \geq \hat{E}(\alpha, m)$ under either selection. $\square$

Lemma A.6. *For each $k \in \mathcal{K}$, let $\delta_m(k) \equiv A(k; m+1) - A(k; m)$. Condition (A49) holds if and only if $\delta_m(k)$ is weakly decreasing in k , that is, $\delta_m(k+1) \leq \delta_m(k)$ for every $k \in \{1, \dots, n-1\}$.*

Proof. By the definition of $G_{k,k'}$ in (6), for every $k < k'$,

$$G_{k,k'}(m+1) - G_{k,k'}(m) = \delta_m(k') - \delta_m(k) = \sum_{j=k}^{k'-1} [\delta_m(j+1) - \delta_m(j)].$$

If every adjacent difference is nonpositive, every such sum is nonpositive, so (A49) holds. Conversely, taking $k' = k+1$ in (A49) gives $\delta_m(k+1) \leq \delta_m(k)$ for every k . $\square$

Proposition A.5 (Neutrality at low reliance on loyalty). *Suppose $1 \leq m \leq n-1$. Define*

$$\alpha_N(n, V) \equiv \min_{1 \leq m \leq n-1} \min_{2 \leq \ell \leq n} \hat{\alpha}_{1,\ell}(m), \quad \hat{\alpha}_{1,\ell}(m) \equiv \frac{E^*(1) - E^*(\ell)}{A(\ell; m) - A(1; m)}. \quad (\text{A53})$$

Then $\alpha_N(n, V) > 0$, and for every $\alpha \in [0, \alpha_N(n, V)]$ and every $1 \leq m \leq n-1$, $k = 1 \in K^*(\alpha, m)$; if $\alpha < \alpha_N(n, V)$, $k^*(\alpha, m) = 1$ is the unique optimum. Authority-capital neutrality therefore holds not merely in the limit $\alpha \rightarrow 0$, but uniformly over the entire interval $[0, \alpha_N(n, V))$, with $\alpha_N(n, V) > 0$ independent of m .

Proof. By Lemma A.4, $A(k; m)$ is nondecreasing in k and strictly increasing on $\{1, \dots, n_d+1\}$, where $n_d = n - m \geq 1$; since $A(1; m) = 1 - 2(n-m)/n < 1$ for $m \geq 1$ and $A(k; m)$ reaches its maximum value of one at $k = n_d + 1 \leq n$, it follows that $A(\ell; m) > A(1; m)$ for every $\ell \geq 2$. Because E^* is strictly decreasing (Proposition A.1), $\hat{\alpha}_{1,\ell}(m)$ in (A53) is therefore well defined, finite, and strictly positive for every $\ell \in \{2, \dots, n\}$ and every $m \in \{1, \dots, n-1\}$. This index set is finite, so $\alpha_N(n, V)$, being the minimum of finitely many strictly positive numbers, is itself strictly positive.

Fix m and $\alpha \leq \alpha_N(n, V)$. There is no $j < 1$ in $\mathcal{K}$, so the lower bound of the global-optimality interval in Remark A.2 is vacuous, and $k = 1$ is optimal if and only if $\alpha \leq \min_{\ell > 1} \hat{\alpha}_{1,\ell}(m)$. Since $\alpha_N(n, V) \leq \hat{\alpha}_{1,\ell}(m)$ for every such ℓ , this holds, so $1 \in K^*(\alpha, m)$. If $\alpha < \alpha_N(n, V)$, the inequality is strict against every $\ell > 1$, so Remark A.2 implies $k = 1$ strictly dominates every $\ell > 1$, giving $K^*(\alpha, m) = \{1\}$. As m was arbitrary, the claim holds for every $1 \leq m \leq n - 1$. $\square$

Proposition A.6 (Substitution at high reliance on loyalty). *Suppose $1 \leq m \leq n - 1$ and let $n_d = n - m$. By Lemma A.4, $A(k; m)$ first attains its maximum value of one at $k^\dagger \equiv n_d + 1 = n - m + 1$. Define, for each $k' \in \{1, \dots, n_d\}$,*

$$\alpha_{k'}^\dagger(m) \equiv \frac{E^*(k') - E^*(n - m + 1)}{1 - A(k'; m)} > 0, \quad \alpha_S(n, V) \equiv \max_{1 \leq m \leq n-1} \max_{1 \leq k' \leq n-m} \alpha_{k'}^\dagger(m) < \infty. \quad (\text{A54})$$

Then for every $\alpha > \alpha_S(n, V)$ and every $1 \leq m \leq n - 1$,

$$k^*(\alpha, m) = n - m + 1, \quad (\text{A55})$$

which is strictly decreasing in m . Consequently, for sufficiently weak legal enforcement, authority capital and promotion discretion are global substitutes: a marginal increase in m strictly tightens the optimal eligibility rule and strictly raises equilibrium output $\hat{E}(\alpha, m) = E^*(n - m + 1)$.

Proof. Write $k^\dagger \equiv n - m + 1$. For every $k'' > k^\dagger$, Lemma A.4 gives $A(k''; m) = A(k^\dagger; m) = 1$, so

$$U_S(k^\dagger; \alpha, m) - U_S(k''; \alpha, m) = E^*(k^\dagger) - E^*(k'') > 0$$

for every $\alpha \geq 0$, by strict monotonicity of E^* (Proposition A.1); $k^\dagger$ therefore strictly dominates every larger rule regardless of α .

For $k' \leq n_d$, $A(k'; m) < 1 = A(k^\dagger; m)$, so $\alpha_{k'}^\dagger(m)$ in (A54) coincides with the pairwise cutoff $\hat{\alpha}_{k', k^\dagger}(m)$ of Remark A.2, and $k^\dagger$ weakly dominates k' once $\alpha \geq \alpha_{k'}^\dagger(m)$, strictly once $\alpha > \alpha_{k'}^\dagger(m)$. Since $\alpha_S(n, V) \geq \alpha_{k'}^\dagger(m)$ for every $k' \leq n_d$ and every m , any $\alpha > \alpha_S(n, V)$ has $k^\dagger$ strictly dominating every $k' < k^\dagger$ as well as every $k'' > k^\dagger$, so $K^*(\alpha, m) = \{n - m + 1\}$. Because $n_d = n - m$ is strictly decreasing in m , so is $k^\dagger = n - m + 1$; the output claim follows from $\hat{E}(\alpha, m) \equiv E^*(k^*(\alpha, m))$ together with strict monotonicity of E^* . $\square$

Between the two thresholds identified above, $\alpha \in (\alpha_N(n, V), \alpha_S(n, V)]$, the optimal rule need not be monotone in m unless condition (A49) holds. The following counterexample shows that the condition can fail.

Proposition A.7 (Counterexample to monotonicity in m). *The optimal competition rule need not be monotone in authority capital m . In particular, let $n = 5$, $V = 1$, and*

$\alpha = 1$. Then

$$k^*(1, 1) = 1, \quad k^*(1, 2) = 2, \quad k^*(1, 3) = 1. \quad (\text{A56})$$

Proof. Using (A41) and Lemma A.4, the relevant payoffs are as follows. When $m = 1$,

$$\begin{aligned} U_S(1; 1, 1) &= 1.4, & U_S(2; 1, 1) &= \sqrt{\frac{3}{2}} \approx 1.2247, \\ U_S(3; 1, 1) &\approx 1.2451, & U_S(4; 1, 1) &= 1.25, & U_S(5; 1, 1) &= 1, \end{aligned} \quad (\text{A57})$$

so $k^*(1, 1) = 1$. When $m = 2$,

$$U_S(1; 1, 2) = 1.8, \quad U_S(2; 1, 2) = \sqrt{\frac{3}{2}} + \frac{5}{8} \approx 1.8497, \quad (\text{A58})$$

and the remaining rules yield smaller payoffs, so $k^*(1, 2) = 2$. When $m = 3$,

$$U_S(1; 1, 3) = 2.2, \quad U_S(2; 1, 3) = \sqrt{\frac{3}{2}} + \frac{13}{14} \approx 2.1533, \quad (\text{A59})$$

and the remaining rules again yield smaller payoffs, so $k^*(1, 3) = 1$. Therefore k^* moves from 1 to 2 and then back to 1 as m rises, which contradicts both weakly increasing and weakly decreasing global monotonicity. $\square$

Authority capital thus affects the superior's optimal competition rule through two opposing margins. At intermediate levels, a larger loyal group can increase the marginal political return to relaxing performance competition; at high levels, it can make strict competition compatible with a high probability of a loyal winner. In the counterexample, $\delta_1(1) = A(1; 2) - A(1; 1) = 0.4$ while $\delta_1(2) = A(2; 2) - A(2; 1) = 0.625$, so the increment in authority value is increasing in k and condition (A49) fails. This is precisely where discretion activation arises. Whenever the condition holds, Proposition A.4 guarantees that greater authority capital weakly tightens the optimal rule.

Productivity reversals across environments

This subsection proves Proposition 4, restated below with the explicit construction.

Proposition A.8 (Higher authority reliance can sustain higher output). *For every $n \geq 3$ and $V > 0$, there exist two baseline environments (α_L, m_L) and (α_H, m_H) satisfying*

$$\alpha_H > \alpha_L \quad \text{and} \quad m_H > m_L,$$

such that

$$k^*(\alpha_H, m_H) < k^*(\alpha_L, m_L) \quad \text{and consequently} \quad \widehat{E}(\alpha_H, m_H) > \widehat{E}(\alpha_L, m_L).$$

Thus, although a larger α weakly relaxes competition conditional on a fixed level of

authority capital, legal weakness does not impose an unconditional ordering on productivity across environments with different stocks of authority capital.

Proof. Recall $E^*(k) = \sqrt{V(n-k)/k}$ and $A(k; m) = 1 - 2P_D^*(k; m)$, so that $U_S(k; \alpha, m) = E^*(k) + \alpha A(k; m)$ by (A41).

We construct two environments with $m_L = 1$ and $m_H = n - 1$.

Consider first the low-authority-capital environment $m_L = 1$. Under the strict tournament $k = 1$, $P_D^*(1; 1) = \frac{n-1}{n}$, and hence

$$A(1; 1) = 1 - \frac{2(n-1)}{n} = \frac{2-n}{n}.$$

Now consider the weaker competition rule $k = n - 1$. Since $m = 1$, the number of disloyal subordinates is $n_d = n - 1$, and the closed-form winning probability gives $P_D^*(n-1; 1) = \frac{1}{2(n-1)}$. Therefore,

$$A(n-1; 1) = 1 - \frac{1}{n-1} = \frac{n-2}{n-1}.$$

The superior prefers $k = n - 1$ to $k = 1$ whenever

$$E^*(n-1) + \alpha A(n-1; 1) > E^*(1) + \alpha A(1; 1).$$

Equivalently, $\alpha > \hat{\alpha}_L(n, V)$, where

$$\hat{\alpha}_L(n, V) \equiv \frac{E^*(1) - E^*(n-1)}{A(n-1; 1) - A(1; 1)}.$$

Using $E^*(1) = \sqrt{V(n-1)}$ and $E^*(n-1) = \sqrt{V/(n-1)}$, we obtain

$$E^*(1) - E^*(n-1) = \sqrt{V} \frac{n-2}{\sqrt{n-1}}.$$

Moreover,

$$A(n-1; 1) - A(1; 1) = \frac{n-2}{n-1} + \frac{n-2}{n} = \frac{(n-2)(2n-1)}{n(n-1)}.$$

Hence

$$\hat{\alpha}_L(n, V) = \frac{n\sqrt{V(n-1)}}{2n-1}.$$

It follows that, for every $\alpha_L > \hat{\alpha}_L(n, V)$, the strict tournament $k = 1$ is not optimal when $m_L = 1$, because $k = n - 1$ yields a strictly higher payoff. Therefore, $k^*(\alpha_L, 1) > 1$.

Now consider the high-authority-capital environment $m_H = n - 1$. There is only one disloyal subordinate. Under $k = 1$, $P_D^*(1; n-1) = \frac{1}{n}$, so $A(1; n-1) = 1 - \frac{2}{n}$. For

every $k \geq 2$, the eligible set necessarily contains at least one loyal subordinate. Thus $P_D^*(k; n-1) = 0$ and $A(k; n-1) = 1$.

Among all $k \geq 2$, the rule $k = 2$ produces the highest aggregate output because $E^*(k)$ is strictly decreasing in k . Therefore, to establish that $k = 1$ is globally optimal, it is sufficient to compare $k = 1$ with $k = 2$.

The superior prefers $k = 1$ to $k = 2$ whenever

$$E^*(1) + \alpha A(1; n-1) > E^*(2) + \alpha A(2; n-1).$$

Equivalently, $\alpha < \hat{\alpha}_H(n, V)$, where

$$\hat{\alpha}_H(n, V) \equiv \frac{E^*(1) - E^*(2)}{A(2; n-1) - A(1; n-1)}.$$

Since

$$A(2; n-1) - A(1; n-1) = 1 - \left(1 - \frac{2}{n}\right) = \frac{2}{n},$$

we obtain

$$\hat{\alpha}_H(n, V) = \frac{n}{2} \left[\sqrt{V(n-1)} - \sqrt{\frac{V(n-2)}{2}} \right].$$

We next show that the interval $(\hat{\alpha}_L(n, V), \hat{\alpha}_H(n, V))$ is nonempty. It is sufficient to prove $\hat{\alpha}_H(n, V) > \hat{\alpha}_L(n, V)$. After dividing both sides by the positive term $n\sqrt{V}$, this inequality becomes

$$\frac{1}{2} \left[\sqrt{n-1} - \sqrt{\frac{n-2}{2}} \right] > \frac{\sqrt{n-1}}{2n-1}.$$

Equivalently,

$$(2n-3)\sqrt{n-1} > (2n-1)\sqrt{\frac{n-2}{2}}.$$

Both sides are nonnegative for $n \geq 3$, so squaring preserves the inequality. The resulting condition is

$$2(2n-3)^2(n-1) > (2n-1)^2(n-2).$$

The difference between the left-hand side and the right-hand side is

$$F(n) = 4n^3 - 20n^2 + 33n - 16.$$

At $n = 3$, $F(3) = 11 > 0$. Moreover,

$$F(n+1) - F(n) = 12n^2 - 28n + 17 > 0$$

for every integer $n \geq 3$. Hence $F(n) > 0$ for every $n \geq 3$, establishing $\hat{\alpha}_H(n, V) > \hat{\alpha}_L(n, V)$.

Because the interval is nonempty, choose any $\hat{\alpha}_L(n, V) < \alpha_L < \alpha_H < \hat{\alpha}_H(n, V)$.

In the low-authority-capital environment $(\alpha_L, 1)$, $k^*(\alpha_L, 1) > 1$. In the high-authority-capital environment $(\alpha_H, n - 1)$, $k^*(\alpha_H, n - 1) = 1$. Therefore,

$$k^*(\alpha_H, n - 1) < k^*(\alpha_L, 1),$$

even though $\alpha_H > \alpha_L$.

Finally, because aggregate equilibrium output $E^*(k) = \sqrt{V(n - k)/k}$ is strictly decreasing in k , it follows that $\hat{E}(\alpha_H, n - 1) > \hat{E}(\alpha_L, 1)$. This proves the result. $\square$

Authority Capital and Institutional Change

This section proves Propositions 6, 7, and 8.

Recall that the superior's payoff under competition rule k is

$$U_S(k; \alpha, m) = E^*(k) + \alpha A(k; m), \quad (\text{A60})$$

where

$$E^*(k) = \sqrt{\frac{V(n - k)}{k}} \quad (\text{A61})$$

and

$$A(k; m) = 1 - 2P_D^*(k; m). \quad (\text{A62})$$

Let $n_d = n - m$. From Lemma A.4,

$$P_D^*(k; m) = \begin{cases} \frac{n - k}{R_k(m) [k + n_d R_k(m)]}, & k \leq n_d, \\ 0, & k > n_d, \end{cases} \quad (\text{A63})$$

where

$$R_k(m) = \frac{\binom{n-1}{k}}{\binom{n_d}{k}}. \quad (\text{A64})$$

We first establish how authority capital affects the authority value associated with any fixed competition rule.

Lemma A.7 (Authority Capital and Authority Value). *For every fixed k ,*

$$A(k; m + 1) \geq A(k; m). \quad (\text{A65})$$

Whenever both sides lie in the region in which a disloyal winner occurs with positive probability, the inequality is strict.

Proof. Consider first $k \leq n_d - 1$. Increasing authority capital from m to $m + 1$ reduces

the number of disloyal subordinates from n_d to $n_d - 1$. From (A64),

$$R_k(m+1) = \frac{\binom{n-1}{k}}{\binom{n_d-1}{k}} > \frac{\binom{n-1}{k}}{\binom{n_d}{k}} = R_k(m). \quad (\text{A66})$$

Moreover,

$$\frac{(n_d - 1)R_k(m+1)}{n_d R_k(m)} = \frac{n_d - 1}{n_d - k} \geq 1. \quad (\text{A67})$$

Hence both $R_k(m)$ and $n_d R_k(m)$ weakly increase when m rises. The denominator in (A63) therefore increases, while its numerator $n - k$ is unchanged. Thus

$$P_D^*(k; m+1) < P_D^*(k; m). \quad (\text{A68})$$

If $k = n_d$, then after increasing m by one we have $k > n_d - 1$, so

$$P_D^*(k; m+1) = 0 < P_D^*(k; m). \quad (\text{A69})$$

If $k > n_d$, the probability of a disloyal winner is already zero and remains zero.

Therefore $P_D^*(k; m)$ is weakly decreasing in m . Since

$$A(k; m) = 1 - 2P_D^*(k; m), \quad (\text{A70})$$

it follows that

$$A(k; m+1) \geq A(k; m). \quad (\text{A71})$$

□

Proof of Proposition 6

At $\alpha = 0$, the superior maximizes $E^*(k)$ alone. Because $E^*(k)$ is strictly decreasing in k ,

$$W(0; m) = E^*(1). \quad (\text{A72})$$

For any rule k satisfying $A(k; m) > 0$, the value of α at which the payoff from that rule reaches the strong-law benchmark solves

$$E^*(k) + \alpha A(k; m) = E^*(1). \quad (\text{A73})$$

Define the rule-specific crossing point

$$\tau_k(m) = \frac{E^*(1) - E^*(k)}{A(k; m)} \quad \text{if } A(k; m) > 0, \quad (\text{A74})$$

and set

$$\tau_k(m) = +\infty \quad \text{if } A(k; m) \leq 0. \quad (\text{A75})$$

Because the superior can choose any k , the first point at which some feasible rule can outperform the strong-law benchmark is

$$\alpha^{TP}(m) = \min_{k \in \mathcal{K}} \tau_k(m) = \min_{k: A(k; m) > 0} \frac{E^*(1) - E^*(k)}{A(k; m)}. \quad (\text{A76})$$

For every fixed k , the numerator

$$E^*(1) - E^*(k) \quad (\text{A77})$$

is independent of m . By Lemma A.7, the denominator $A(k; m)$ is weakly increasing in m . Hence, whenever $A(k; m) > 0$,

$$\tau_k(m+1) \leq \tau_k(m). \quad (\text{A78})$$

If $A(k; m) \leq 0$ but $A(k; m+1) > 0$, the rule-specific threshold falls from $+\infty$ to a finite value. Thus $\tau_k(m)$ is weakly decreasing in m for every k under the stated convention.

Taking the minimum across rules preserves weak monotonicity:

$$\alpha^{TP}(m+1) = \min_k \tau_k(m+1) \leq \min_k \tau_k(m) = \alpha^{TP}(m). \quad (\text{A79})$$

This proves Proposition 6. □

Proof of Proposition 7

Fix $m < n/2$.

For every rule k , define

$$L_k(\alpha) = E^*(k) + \alpha A(k; m). \quad (\text{A80})$$

Each $L_k(\alpha)$ is affine in α , and

$$W(\alpha; m) = \max_{k \in \mathcal{K}} L_k(\alpha). \quad (\text{A81})$$

The pointwise maximum of finitely many affine functions is continuous, convex, and piecewise linear. Hence $W(\alpha; m)$ has these properties.

We first establish the behavior of W near $\alpha = 0$. Because $E^*(k)$ is strictly decreasing in k ,

$$E^*(1) > E^*(k) \quad \text{for every } k > 1. \quad (\text{A82})$$

Thus $k = 1$ is uniquely optimal at $\alpha = 0$. Since the choice set is finite, this strict ranking persists for all sufficiently small $\alpha > 0$. Therefore there exists $\varepsilon > 0$ such that

$$W(\alpha; m) = E^*(1) + \alpha A(1; m) \quad \text{for all } \alpha \in [0, \varepsilon). \quad (\text{A83})$$

Under the strict tournament,

$$P_D^*(1; m) = \frac{n - m}{n}, \quad (\text{A84})$$

and therefore

$$A(1; m) = 1 - \frac{2(n - m)}{n} = \frac{2m}{n} - 1. \quad (\text{A85})$$

Since $m < n/2$,

$$A(1; m) < 0. \quad (\text{A86})$$

Equation (A83) therefore implies that $W(\alpha; m)$ is strictly decreasing for all sufficiently small positive α . In particular,

$$W(\alpha; m) < W(0; m) \quad (\text{A87})$$

for sufficiently small $\alpha > 0$.

We next establish that W eventually increases without bound. Let

$$k^\dagger = n - m + 1 = n_d + 1. \quad (\text{A88})$$

Since $k^\dagger > n_d$, a disloyal subordinate cannot win under this rule. Hence

$$P_D^*(k^\dagger; m) = 0 \quad (\text{A89})$$

and

$$A(k^\dagger; m) = 1. \quad (\text{A90})$$

The payoff generated by this feasible rule is therefore

$$L_{k^\dagger}(\alpha) = E^*(k^\dagger) + \alpha. \quad (\text{A91})$$

Consequently,

$$W(\alpha; m) \geq E^*(k^\dagger) + \alpha \longrightarrow +\infty \quad \text{as } \alpha \rightarrow \infty. \quad (\text{A92})$$

Because W is continuous, initially decreasing, and tends to infinity, it attains a global minimum at some positive value of α . Thus

$$\mathcal{M}(m) = \arg \min_{\alpha \geq 0} W(\alpha; m) \quad (\text{A93})$$

is nonempty. Convexity implies that $\mathcal{M}(m)$ is a closed interval, possibly a singleton. Define its endpoints as

$$\underline{\alpha}^{\min}(m) = \min \mathcal{M}(m), \quad \bar{\alpha}^{\min}(m) = \max \mathcal{M}(m). \quad (\text{A94})$$

The initial strict decline implies

$$0 < \underline{\alpha}^{\min}(m) \leq \bar{\alpha}^{\min}(m). \quad (\text{A95})$$

Moreover, because W falls strictly below its value at zero before reaching its minimum,

$$W(\alpha; m) < W(0; m) \quad \text{for every } \alpha \in \mathcal{M}(m). \quad (\text{A96})$$

By (A92), W must eventually exceed $W(0; m)$. Continuity therefore implies that, after leaving the minimizing set, the payoff crosses the strong-law benchmark.

By the definition of $\alpha^{TP}(m)$,

$$W(\alpha^{TP}(m); m) = W(0; m), \quad (\text{A97})$$

and

$$\bar{\alpha}^{\min}(m) < \alpha^{TP}(m). \quad (\text{A98})$$

Combining (A95) and (A98) gives

$$0 < \underline{\alpha}^{\min}(m) \leq \bar{\alpha}^{\min}(m) < \alpha^{TP}(m). \quad (\text{A99})$$

Finally, convexity implies that W is weakly decreasing before its minimizing set and weakly increasing after it. Because the minimizing set has been defined to include every point at which the minimum is attained, W is strictly increasing once $\alpha > \bar{\alpha}^{\min}(m)$. Hence, for

$$\bar{\alpha}^{\min}(m) < \alpha < \alpha^{TP}(m), \quad (\text{A100})$$

further weakening raises the superior's payoff while

$$W(\alpha; m) < W(0; m). \quad (\text{A101})$$

For

$$\alpha > \alpha^{TP}(m), \quad (\text{A102})$$

we instead have

$$W(\alpha; m) > W(0; m). \quad (\text{A103})$$

This establishes all parts of Proposition 7. $\square$

Proof of Proposition 8

Under the strict tournament $k = 1$,

$$P_D^*(1; m) = \frac{n - m}{n}, \quad (\text{A104})$$

so

$$A(1; m) = 1 - 2P_D^*(1; m) = \frac{2m}{n} - 1. \quad (\text{A105})$$

Suppose first that

$$m > \frac{n}{2}. \quad (\text{A106})$$

Then

$$A(1; m) > 0. \quad (\text{A107})$$

For every $\alpha > 0$, the payoff from retaining the strict tournament is therefore

$$U_S(1; \alpha, m) = E^*(1) + \alpha A(1; m) \quad (\text{A108})$$

$$> E^*(1) \quad (\text{A109})$$

$$= W(0; m). \quad (\text{A110})$$

Since $W(\alpha; m)$ is the maximum payoff across all feasible rules,

$$W(\alpha; m) \geq U_S(1; \alpha, m) > W(0; m) \quad (\text{A111})$$

for every $\alpha > 0$. It follows immediately that

$$\alpha^{TP}(m) = 0. \quad (\text{A112})$$

Now consider the boundary case

$$m = \frac{n}{2}. \quad (\text{A113})$$

Then

$$A(1; m) = 0, \quad (\text{A114})$$

so

$$U_S(1; \alpha, m) = E^*(1) = W(0; m) \quad (\text{A115})$$

for every α for which $k = 1$ remains optimal. Since $k = 1$ is uniquely optimal at $\alpha = 0$, continuity and the finiteness of the rule set imply that it remains optimal over a nonempty interval beginning at zero. The optimized payoff is therefore locally flat at the strong-law benchmark.

Thus the sign of the superior's local payoff response at the strong-law benchmark is determined by

$$A(1; m) = \frac{2m}{n} - 1 : \tag{A116}$$

it is negative when $m < n/2$, zero when $m = n/2$, and positive when $m > n/2$. This establishes Proposition 8. $\square$

*

References

- Acemoglu, D., S. Johnson, and J. A. Robinson (2005). Institutions as the fundamental cause of long-run growth. In P. Aghion and S. N. Durlauf (Eds.), *Handbook of Economic Growth*, Volume 1A, pp. 385–472. Amsterdam: Elsevier.
- Acemoglu, D. and J. A. Robinson (2002). The political economy of the Kuznets curve. *Review of Development Economics* 6(2), 183–203.
- Baker, G., R. Gibbons, and K. J. Murphy (1994). Subjective performance measures in optimal incentive contracts. *The Quarterly Journal of Economics* 109(4), 1125–1156.
- Bermeo, N. (2016). On democratic backsliding. *Journal of Democracy* 27(1), 5–19.
- Berry, S. K. (1993). Rent-seeking with multiple winners. *Public Choice* 77(2), 437–443.
- Bowles, S. and H. Gintis (1993). The revenge of homo economicus: Contested exchange and the revival of political economy. *Journal of Economic Perspectives* 7(1), 83–102.
- Bueno de Mesquita, B., A. Smith, R. M. Siverson, and J. D. Morrow (2003). *The Logic of Political Survival*. Cambridge, MA: MIT Press.
- Burawoy, M. (1985). *The Politics of Production: Factory Regimes Under Capitalism and Socialism*. London: Verso.
- Capoccia, G. and R. D. Kelemen (2007). The study of critical junctures: Theory, narrative, and counterfactuals in historical institutionalism. *World Politics* 59(3), 341–369.
- Coase, R. H. (1937). The nature of the firm. *Economica* 4(16), 386–405.
- Collier, R. B. and D. Collier (1991). *Shaping the Political Arena: Critical Junctures, the Labor Movement, and Regime Dynamics in Latin America*. Princeton: Princeton University Press.
- Dessein, W. and L. Garicano (2025). La cordata: Loyalty in political tournaments. Technical report, Working Paper.
- Egorov, G. and K. Sonin (2011). Dictators and their viziers: Endogenizing the loyalty–competence trade-off. *Journal of the European Economic Association* 9(5), 903–930.

- Gibbons, R. and K. J. Murphy (1992). Optimal incentive contracts in the presence of career concerns: Theory and evidence. *Journal of Political Economy* 100(3), 468–505.
- Greif, A. and D. D. Laitin (2004). A theory of endogenous institutional change. *American Political Science Review* 98(4), 633–652.
- Grillo, E., Z. Luo, M. Nalepa, and C. Prato (2024). Theories of democratic backsliding. *Annual Review of Political Science* 27, 381–400.
- Helmke, G. and S. Levitsky (2004). Informal institutions and comparative politics: A research agenda. *Perspectives on Politics* 2(4), 725–740.
- Kuznets, S. (1955). Economic growth and income inequality. *American Economic Review* 45(1), 1–28.
- Kuznets, S. (1966). *Modern Economic Growth: Rate, Structure, and Spread*. New Haven: Yale University Press.
- Lazear, E. P. and S. Rosen (1981, October). Rank-order tournaments as optimum labor contracts. *Journal of Political Economy* 89(5), 841–864.
- Mahoney, J. and K. Thelen (2010). A theory of gradual institutional change. In J. Mahoney and K. Thelen (Eds.), *Explaining Institutional Change: Ambiguity, Agency, and Power*, pp. 1–37. Cambridge: Cambridge University Press.
- Miller, G. J. (1992). *Managerial Dilemmas: The Political Economy of Hierarchy*. Political Economy of Institutions and Decisions. Cambridge: Cambridge University Press.
- Nalebuff, B. J. and J. E. Stiglitz (1983). Information, competition, and markets. *The American Economic Review* 73(2), 278–283.
- Persson, T., G. Roland, and G. Tabellini (2000). Comparative politics and public finance. *Journal of Political Economy* 108(6), 1121–1161.
- Prendergast, C. (1999). The provision of incentives in firms. *Journal of Economic Literature* 37(1), 7–63.
- Rai, B. K. and R. Sarin (2009, July). Generalized contest success functions. *Economic Theory* 40(1), 139–149.
- Robinson, J. A. and T. Verdier (2013). The political economy of clientelism. *The Scandinavian Journal of Economics* 115(2), 260–291.

Rosen, S. (1986, September). Prizes and incentives in elimination tournaments. *American Economic Review* 76(4), 701–715.

Simon, H. A. (1951). A formal theory of the employment relationship. *Econometrica* 19(3), 293–305.

Skaperdas, S. (1996, June). Contest success functions. *Economic Theory* 7(2), 283–290.

Tullock, G. (1980). Efficient rent seeking. In J. M. Buchanan, R. D. Tollison, and G. Tullock (Eds.), *Toward a Theory of the Rent-Seeking Society*, pp. 97–112. College Station: Texas A&M University Press.

Zakharov, A. V. (2016). The loyalty-competence trade-off in dictatorships and outside options for subordinates. *Journal of Politics* 78(2), 457–466.